MAE 502, Spring 2019 HW3 Solution

Prob 1

(a) Not a Sturm-Liouville system

(b) There is only one eigenvalue, $c = [ln(2+\sqrt{3})]^2 \approx 1.7344$. The associated eigenfunction is $G(x) = \cosh(\sqrt{c}\, x)$.

Prob 2

(a)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\cos(n\pi t)$$

where

$$a_n = 2\int_0^1 [\sin(\sqrt{x} - x)]^2 \sin(n\pi x)\, dx$$

(b)

$$u(x,t) = \sum_{\substack{n=1 \\ <n\ is\ odd>}}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right)\cos\left(\frac{n\pi t}{2}\right)$$

where the summation is carried out over odd values of *n* only, and

$$a_n = 2\int_0^1 [\sin(\sqrt{x} - x)]^2 \sin\left(\frac{n\pi x}{2}\right) dx$$

Prob 3

$$u(x,t) = \sum_{n=-\infty}^{\infty} C_n(0) e^{(inU - n^2K)t + inx}$$

where

$$C_n(0) = \frac{1}{2\pi}\int_0^{2\pi} u(x,0)e^{-inx} dx$$

Prob 4

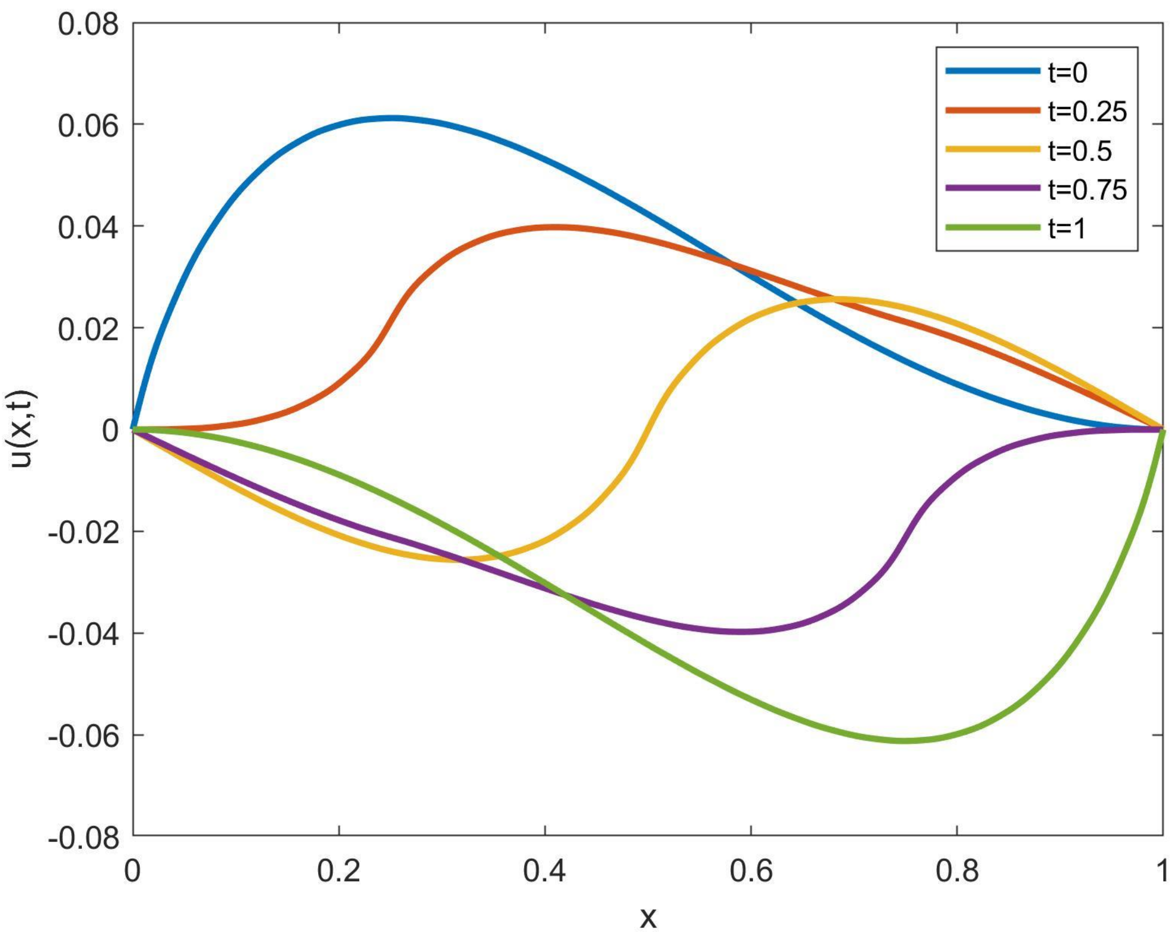
$$u(x,t) = e^t + \sin(x - \frac{t^2}{2})$$

Prob 5

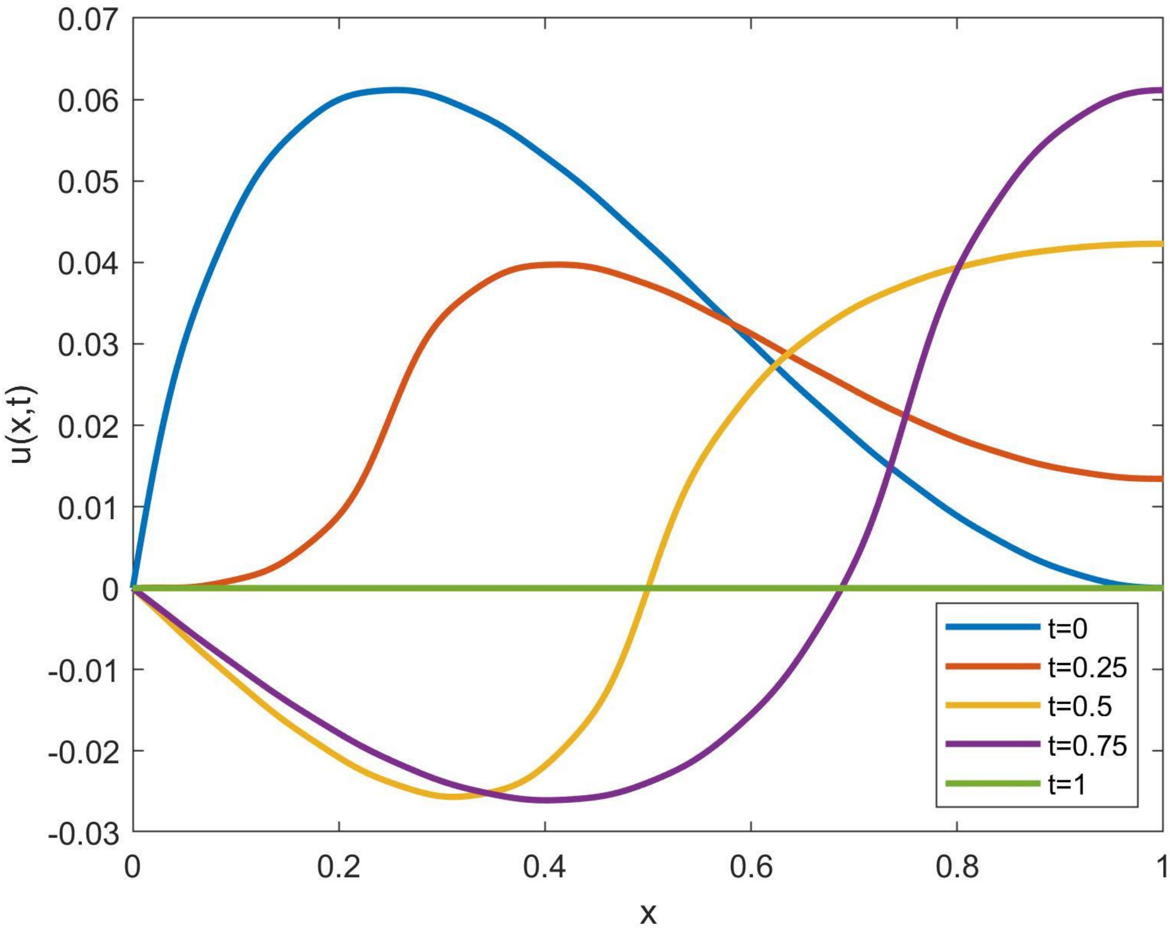
$$u(x,t) = t\sin(x) + \cosh(\sqrt{3}\, t)\cos(2x)$$

See next 3 pages for plots

Plot for Prob 2a



Plot for Prob 2b



Plot for Prob 3