MAE502, Spring 2019 Homework # 4

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration.

For ALL problems in this homework, we expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be real and should be expressed in real numbers and functions.

Prob 1 (2 points)
(a) For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u + \sin(\pi x)\, e^t$$

with the boundary conditions:

(i) $u(0, t) = 0$ (ii) $u(1, t) = 0$ (iii) $u(x, 0) = \sin(\pi x) + \sin(2\pi x)$.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \cos(x)$$

with the boundary conditions,

(i) $u(0, t) = 2$ (ii) $u(\pi, t) = 0$ (iii) $u(x, 0) = 1 + \cos(x) + \sin(x)$.

Prob 3 (2 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + u + 3 + \sin(x)e^{-t}$$

with periodic boundary conditions in the x-direction, and the boundary conditions in the t-direction given as

(i) $u(x, 0) = 1 + \sin(3x)$ (iv) $u_t(x, 0) = \cos(x)$.

Prob 4 (1 point)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$$

with the boundary conditions,

(i) $u(0, t) = 1$ (ii) $u_x(\pi, t) = 1$ (iii) $u(x, 0) = \cos(x) - \sin(x)$.