MAE502, Spring 2019 Homework #5

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration.

For ALL problems in this homework, we expect a closed-form solution without any unevaluated integrals. The solution should be real and should be expressed in real numbers and functions.

You might find the following formulas useful:

$$\int_0^\infty \frac{\cos(bx)}{1+x^2}dx = \left(\frac{\pi}{2}\right)\mathrm{e}^{-|b|}$$, where $|b|$ is the absolute value of b.

$$\int_0^\infty \mathrm{e}^{-x}\cos(bx)dx = \frac{1}{1+b^2}$$

$$\int_0^\infty \mathrm{e}^{-x^2}\cos(2bx)dx = \frac{\sqrt{\pi}}{2}\mathrm{e}^{-b^2}$$

Task 1 (3 points)

(a) For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = 3\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}$$

with the boundary condition,

$$u(x,0) = e^{-x^2}$$

Plot the solution, $u(x,t)$, as a function of x at $t = 0$ and 1. Please put the two curves in one plot.

(b) Repeat (a) but change the factor of "3" in the PDE to "–3". Plot the solution as a function of x at $t = 0$ and 1.

(c) Repeat (a) but eliminate the second-derivative term from the PDE. In other words, the PDE is simply $\partial u/\partial t = 3\ \partial u/\partial x$. Plot the solution as a function of x at $t = 0$ and 1.

The recommended range for plotting of $u(x,t)$ for Part (a)-(c) is $-10 \leq x \leq 10$.

Task 2 (2 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = (1+t)\frac{\partial u}{\partial x} + e^{-t}u$$

with the boundary condition,

$$u(x,0) = e^{-x^2}$$

Task 3 (2 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, consider the PDE

$$\frac{\partial u}{\partial t} = (2-t)\frac{\partial^2 u}{\partial x^2}$$

with the boundary condition,

$$u(x,0) = \frac{1}{1+x^2}$$

Evaluate $u(x,t)$ at $x = 2$, $t = 4$. (Note: The key deliverable of this task is the exact value of $u(2,4)$. You may or may not need to find the full solution, $u(x, t)$ for all x and t, to answer the key question.)