MAE502, Spring 2019 Homework #6

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration.

For ALL problems in this homework, we expect a closed-form solution without any unevaluated integrals.

Prob 1 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = u$$

with the boundary condition

$$u(x,0) = e^{-x^2}.$$

Prob 2 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + t\frac{\partial u}{\partial x} = 1$$

with the boundary condition,

$$u(x,0) = \begin{cases} -\frac{1}{x}, & if\ x \leq -1 \\ 1, & if\ x > -1 \end{cases}$$

Plot the solution as a function of x at $t = 0.25$ and 0.5, and the initial state, $u(x, 0)$. Collect all 3 curves in one plot.

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 1$$

with the boundary conditions:

(i) $u(x,0) = e^{-x^2}$, (ii) $u_t(x,0) = 2xe^{-x^2}$.

Prob 4 (1 point)
For $u(x, y, t)$ defined on the domain of $-\infty < x < \infty$, $-\infty < y < \infty$, and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (1+t)u$$

with the boundary condition,

$$u(x,y,0) = e^{-(x^2+y^2)}.$$