MAE 561/471 Fall 2013 Homework #0A

This is a mini exercise equivalent to approximately 1/8 of a full homework assignment. This exercise is for refreshing your memory on elementary fluid mechanics as a prerequisite of this course.

No restriction on discussion with peers for this problem.

1. Consider a fluid mechanical system illustrated in Fig. 1. The x and y directions point to the right and top edges of the paper. The z direction is perpendicular to the x-y plane. A fluid of constant density (ρ) and dynamic viscosity (μ) fills a long channel of length L in the x direction and bounded by two walls at $y = 0$ and $y = H$. The fluid and the walls are infinitely deep in the z direction and the system is assumed uniform in that direction, which renders the flow two-dimensional. The fluid obeys no-slip boundary conditions for velocity at the two walls. Starting from rest, a set of device is used to maintain a pressure difference, Δp, between the left (higher pressure) and right (lower pressure) openings of the channel. This imposed pressure difference drives a flow in the positive x direction which reaches a steady state after a sufficiently long time. To simplify the discussion, we further assume that the channel is very long ($L >> H$) such that at the steady state the flow can be considered uniform in the x direction, i.e., $u \equiv u(y)$ where u is the velocity in the x direction. **(a)** Find the velocity profile, $u(y)$, of the steady state and express it as a function of y, L, H, ρ, μ, and Δp. Plot the velocity profile as a function of y using the specific values of (L, H, ρ, μ, Δp) given at the end of Part (b). **(b)** What is the maximum velocity associated with the velocity profile you find in Part (a)? At what location (what value of y) does this maximum velocity occur? Express your answers in terms of the parameters L, H, ρ, μ, and Δp. Evaluate the maximum velocity, in m/s, given $L = 100$ m, $H = 0.1$ m, $\rho = 1000$ kg/m^3, $\mu = 0.001$ (Pa)(s), and $\Delta p = 10$ Pa. (The values of ρ and μ given here are typical of water.) **(c)** If you have solved the problem correctly, you will find that the maximum velocity increases with the width of the channel (H) when other parameters are fixed. Explain, physically, why that's the case.

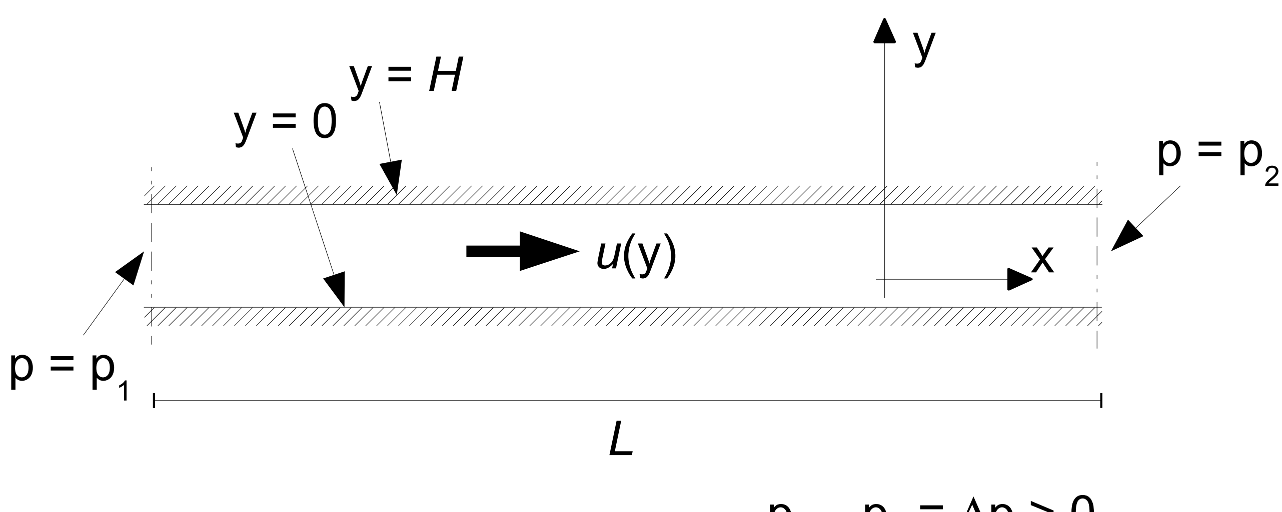

Fig. 1