

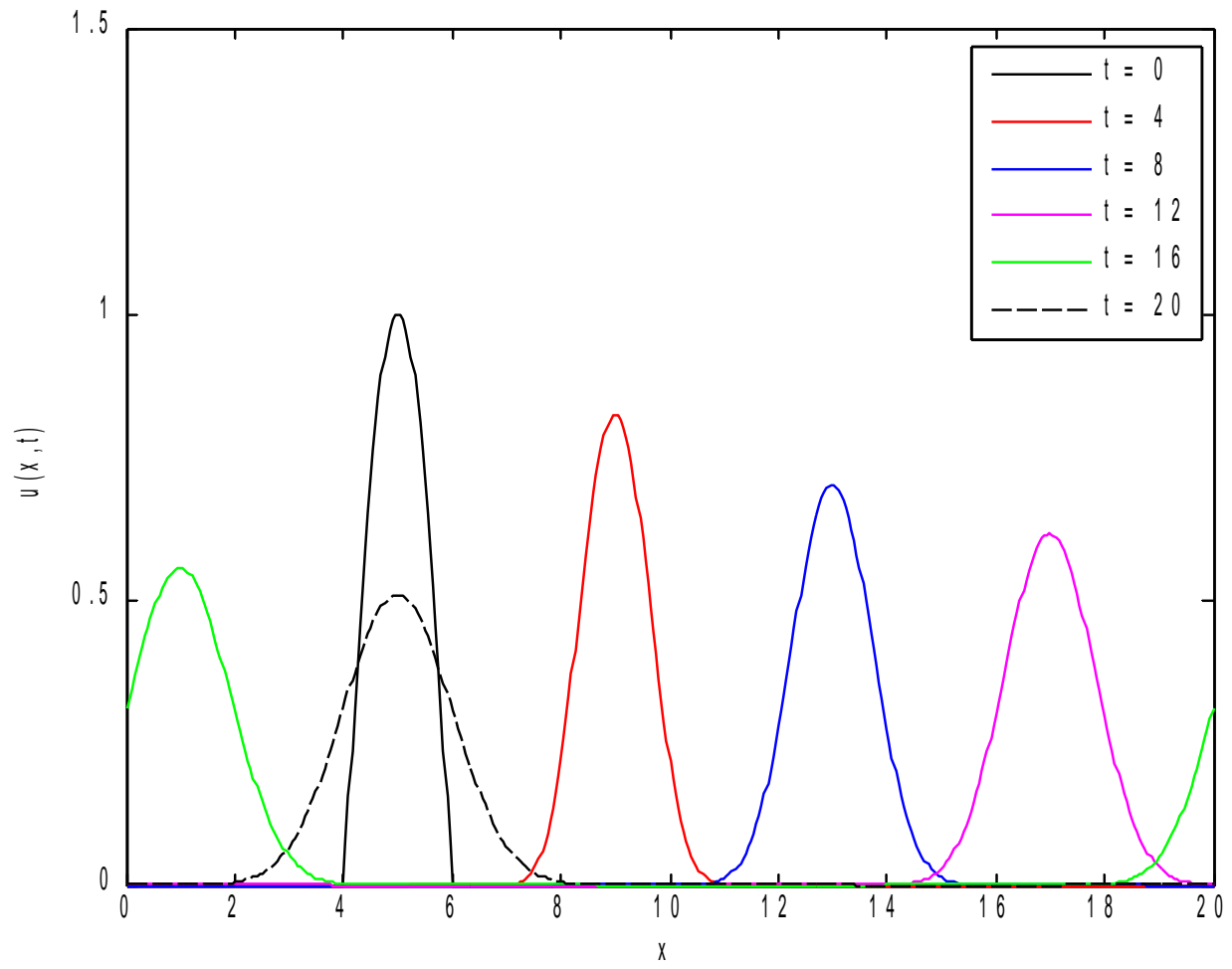
HW1 All solutions prepared by HPH

Prob 1a code

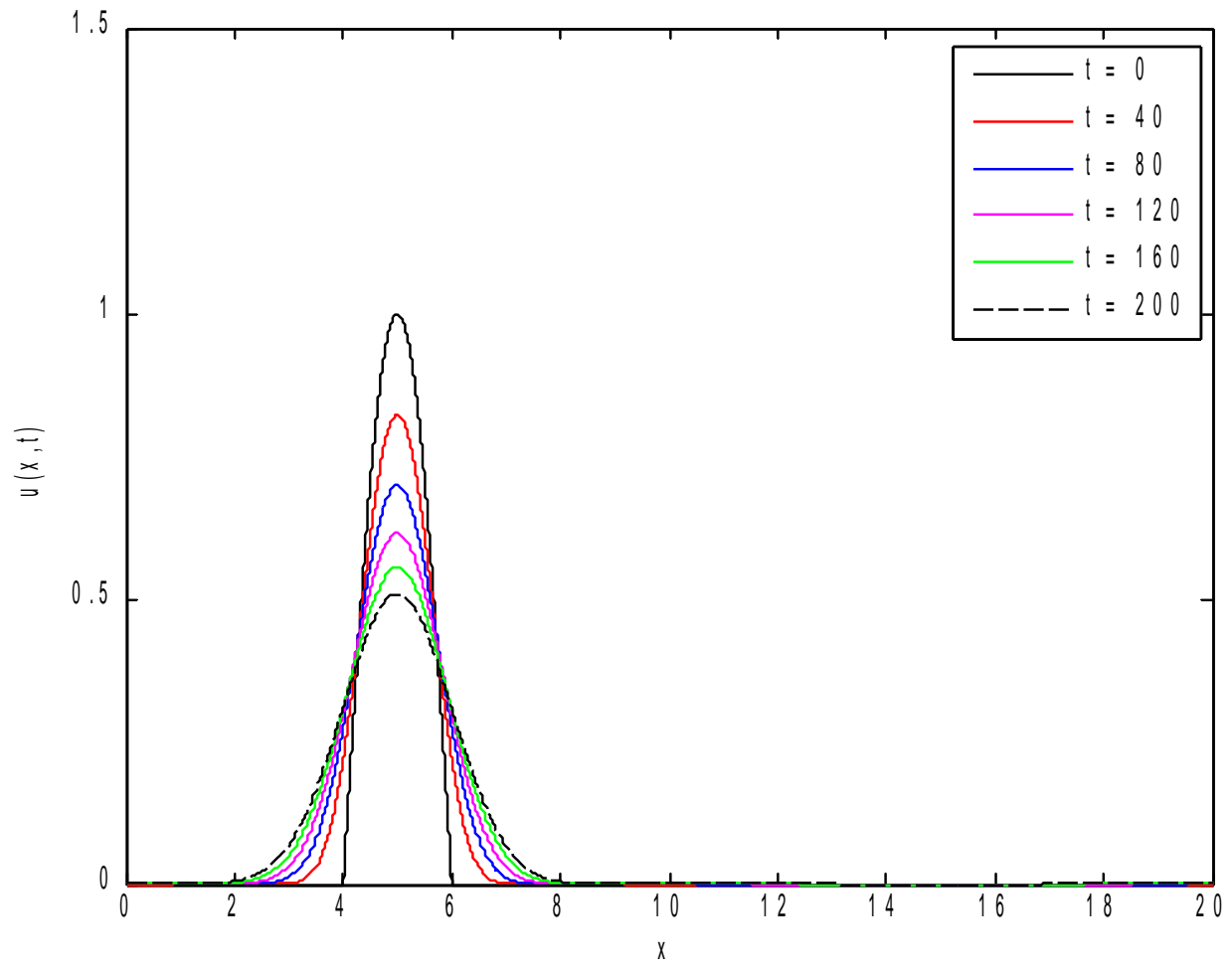
(the codes for the other problems are not provided as they can be easily modified from this one)

```
clear
dx = 0.05; dt = 0.01; C = 1; A1 = 1-C*dt/dx; A2 = C*dt/dx;
x = [0:dx:20];
N = length(x);
for k = 2:N
    if (x(k) >= 4) && (x(k) <= 6)
        u(k) = cos(0.5*pi*(x(k)-5));
    else
        u(k) = 0;
    end
end
u(1) = u(N);
for k = 1:N
    uplot(1,k) = u(k);
end
for iout = 1:5
    for n = 1:400
        for k = 2:N
            u1(k) = A1*u(k)+A2*u(k-1);
        end
        u1(1) = u1(N);
        for k = 1:N
            u(k) = u1(k);
        end
    end
end
for k = 1:N
    uplot(iout+1,k) = u(k);
end
end
plot(x,uplot(1,:), 'k-',x,uplot(2,:), 'r-',x,uplot(3,:), 'b-',...
      x,uplot(4,:), 'm-',x,uplot(5,:), 'g-',x,uplot(6,:), 'k--')
axis([0 20 0 1.5])
legend('t = 0', 't = 4', 't = 8', 't = 12', 't = 16', 't = 20',...
       'Location', 'NorthEast')
xlabel('x');ylabel('u(x,t)')
```

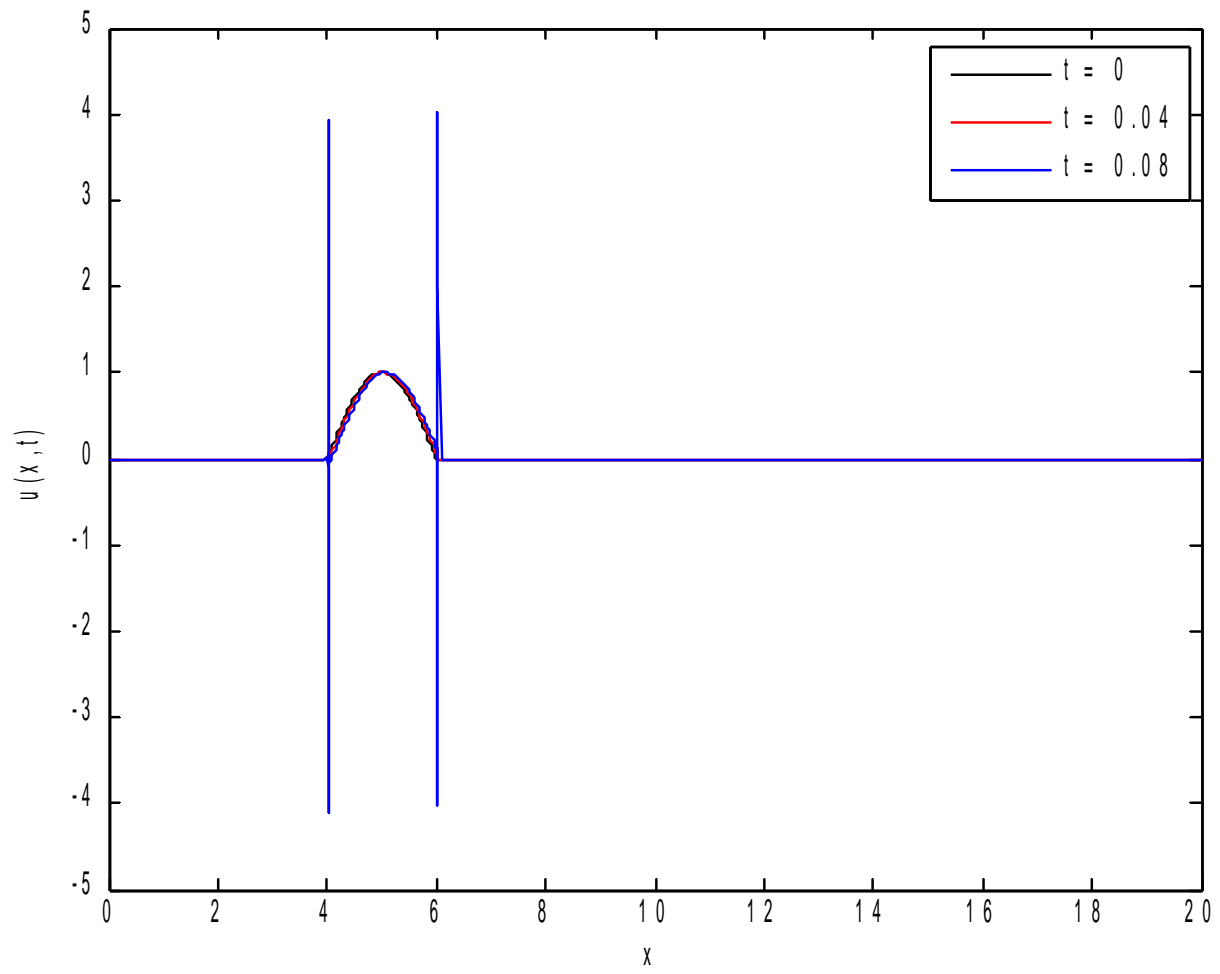
Prob 1a plot



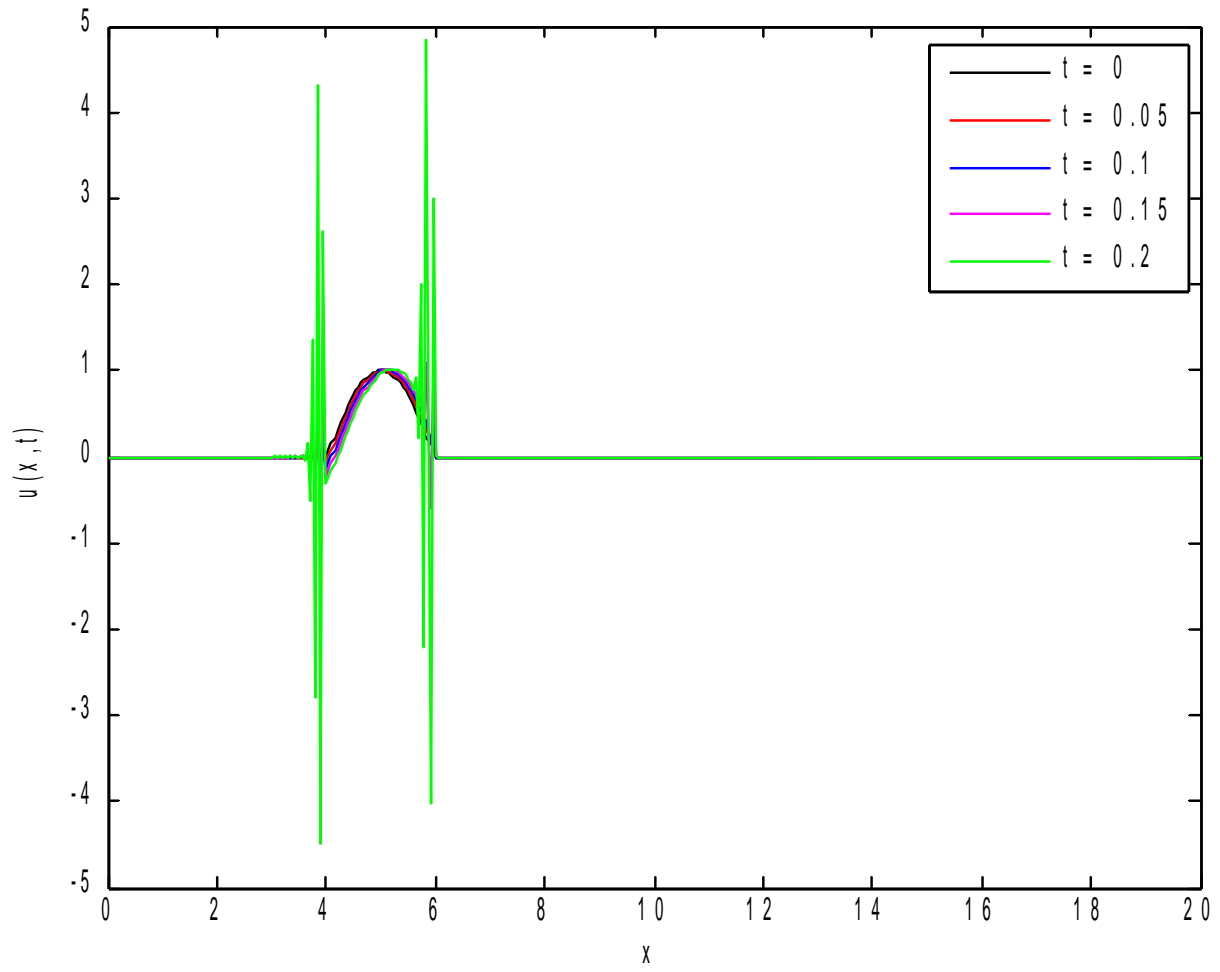
Prob 1b plot



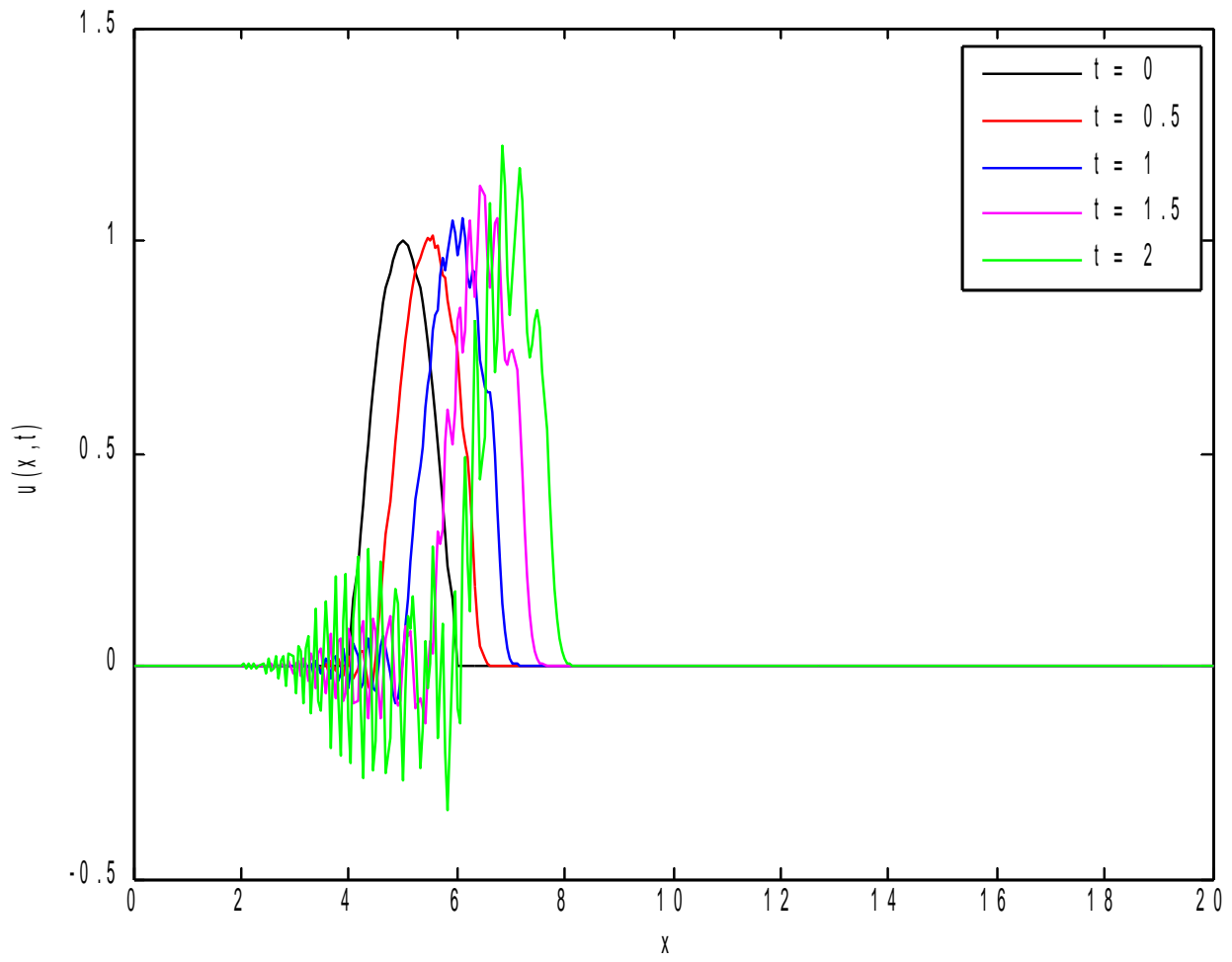
Prob 1c plot



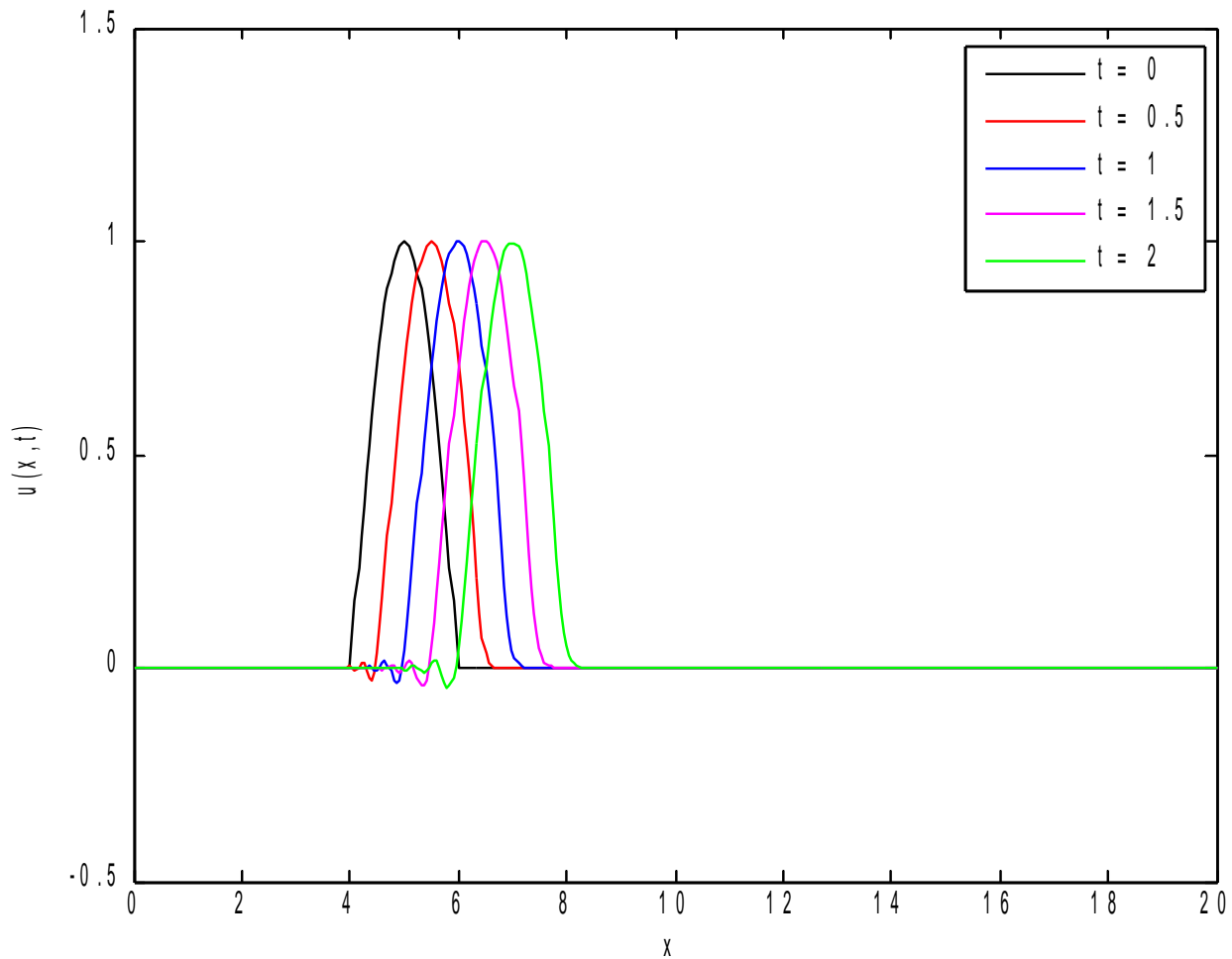
Prob 2 plot



Prob 3 plot



Prob 4 plot



Summary:

Prob1:

- (a) and (b) The upstream (FTBS) scheme works when the numerical stability criterion is satisfied.
- (c) The upstream scheme does not work if the stability criterion is not satisfied.

Prob2:

The downstream (FTFS) scheme never works.

Prob3:

Even though it works for a short time, the FTCS scheme eventually fails because of numerical instability. (This is revisited in Prob 2 of HW3)

Prob4:

The Lax-Wendroff scheme works when the numerical stability criterion is satisfied. Except for producing a potentially undesirable "oscillatory tail", it generally outperforms the 1st-order (FTBS) scheme.