MAE561/471 Fall 2013 HW7

This assignment is for bonus only. The rules of collaboration are the same as those for the final project.

1. *Bonus question*. For $u(x,t)$ and $h(x,t)$ defined on the domain of $0 \le x \le 10$ and $t \ge 0$, consider the following system of equations,

$$\frac{\partial u}{\partial t} = 2\frac{\partial h}{\partial x} \quad , \qquad (1)$$

$$\frac{\partial h}{\partial t} = 0.5\frac{\partial u}{\partial x} \quad , \qquad (2)$$

with the boundary conditions,

$u(0, t) = 0,$

$u(10, t) = 0,$

and the initial conditions,

$h(x,0) \equiv 0,$

$u(x,0) = \mathrm{P}(x),$

where

$$\mathrm{P}(x) \equiv \cos[0.5\,\pi\,(x-5)] \ , \ \text{if } 4 \le x \le 6$$
$$\equiv 0 \qquad\qquad\qquad\quad , \ \text{otherwise.}$$

(Note that the P(x) here is identical to that used in HW1.) Solve the system numerically by using a staggered grid system as illustrated in Fig. 1, and the forward-in-time and central-in-space finite difference scheme for both equations. Use $\Delta x = 0.01$ and your own choice of Δt. Plot the solution at $t = 0.75$ and 2.5, along with the initial state at $t = 0$. See Fig. 1 for the definition of Δx. For this problem, the two boundary points should be the grid points for u (see Fig. 1), such that there is no need to impose the boundary conditions for h. As usual, submit your code. Only high-quality solutions will receive bonus.

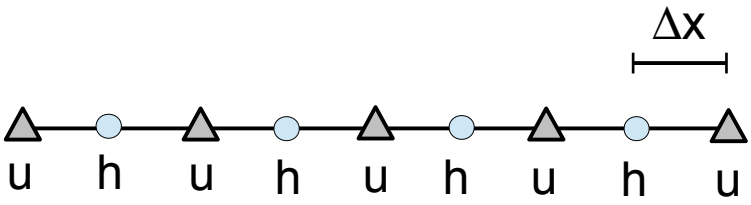

Fig. 1