MAE571, Fall 2014 Homework #1, Revised

Due Wednesday, October 1, before the start of class. Please submit hard copy of the solutions, including the print out of computer codes (in Matlab, Fortran ,C++, etc.) used in the work. Additional rules on collaboration will be released separately. Please always follow the rules.

1. A steady two-dimensional flow is given as

$$u(x, y) = -x + 0.6\,y - 1$$
$$v(x, y) = \quad y + 0.4\,x\,y\,.$$

A blob of fluid initially (at $t = 0$) occupies a circle centered at $(x, y) = (2, 0)$ with a radius of 0.3 (see illustration in Fig. 3 in **Additional Note**). Given the flow field, track the blob to $t = 0.8$ and 2.0 and make a plot of the outline of the blob at $t = 0$, 0.8, and 2.0. Please also superimpose the vectors of the (Eulerian) velocity field in your plot. Hint: It suffices to track the movement of the boundary points of the blob, then connect them to outline the blob at a given time. In Matlab, the "fill" command fills an area enclosed by a set of points. The vector field can be plotted by the "quiver" command in Matlab. (45%)

2. In Prob. 1, will the "dyed blob" of fluid conserve its volume (or "area" since it is 2-D) as it moves and undergoes deformation? If the steady flow in Prob. 1 is replaced by the unsteady flow given as

$$u(x, y) = -x\,t + 0.6\,y$$
$$v(x, y) = \quad y\,t - 0.3\,x\ ,$$

will the dyed blob conserve its volume? Must provide a rigorous reasoning to receive credit. A mere "yes" or "no" answer to each of the two questions will receive zero credit, even if it is correct. (10%)

3. The general form of the vorticity equation for an inviscid flow is given in the textbook (Eq. 1.39) as

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\omega}}{\rho}\right) + \boldsymbol{V}\cdot\nabla\left(\frac{\boldsymbol{\omega}}{\rho}\right) = \left(\frac{\boldsymbol{\omega}}{\rho}\cdot\nabla\right)\boldsymbol{V} - \left(\frac{1}{\rho}\right)\nabla\left(\frac{1}{\rho}\right)\times\nabla p\ \ ,$$

where $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$ is the vorticity vector, $\boldsymbol{V} = (u, v, w)$ is the velocity vector, and ρ and p are density and pressure. Note that we have rewritten the equation in Eulerian form. If the flow is initially at rest and density and pressure are maintained by an external forcing such that they remain steady and depend only on x and z, i.e., $\rho \equiv \rho(x, z)$, $p \equiv p(x, z)$, as shown in Fig. 1 and described in its caption, calculate the initial (at $t = 0$) tendency of the y-component of vorticity, $\partial\omega_y/\partial t$, at the location marked by a star in Fig. 1. Note that since vorticity has the unit of s^{-1}, its tendency in time should have the unit of s^{-2}. (35 %)

Note: You also have the option of dropping Prob 3(a) but just solving Prob 3(b) in the original version. (In that case, the weight of Prob 3(b) will be adjusted to 35%.) The answer will be the same as the revised Prob 3. The reason for the revision is that there were inherent inconsistencies in the original Prob 3(a) which the instructor overlooked. (In short, the statement in the original version that "the circulation will remain in the x-z plane" was not true; At $t > 0$ the v-velocity could be induced by mass continuity.)

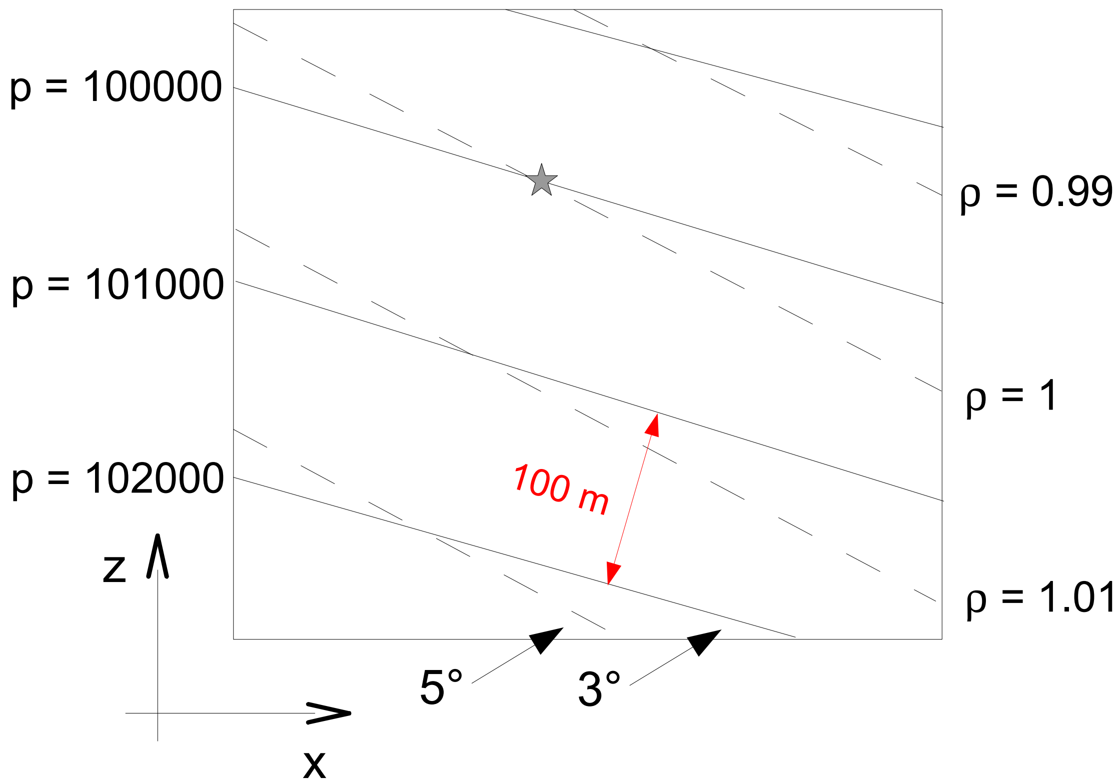

Fig. 1 The density and pressure fields for Prob 3b. Both ρ and *p* are steady and are a function of *x* and *z* only. The *x*- and *z*-axis are as shown, and y-axis is pointing into the paper. It suffices to show the contours of ρ and *p* in the *x*-*z* plane. Although ρ and *p* are not uniform in space, their gradient vectors are uniform and the contours of each of ρ and *p* form a simple set of parallel lines. The contours of ρ (dashed line) intersect with the horizontal line at a 5° angle while those of *p* (solid line) intersect with the horizontal line at a 3° angle. Selected contours are labeled. The units of ρ and *p* are kg/m^3 and Pa. The contours shown are with a uniform 100 m spacing. Along the direction normal to the contours of ρ, density changes by 0.01 kg/m^3 per 100 m. Along the direction normal to the contours of *p*, pressure changes by 1000 Pa per 100 m.

4. A field campaign was conducted to measure the variation of temperature across the city of Mesa. Two teams were deployed to measure the air temperature along University Drive which runs in the east-west direction through the city. The first team consisted of local volunteers whose houses happen to be located on University Dr. Thermometers were set up in their front yards (illustrated in Fig. 2 as the line of triangles) to monitor the local temperature. The second team operated a mobile unit (see illustration) with a thermometer on board a truck that moved eastward along University Dr. at a speed of 5 m/s. The 1st team reported that at any given time during the campaign temperature decreases eastward along University Dr. with a constant rate of 0.1 °C/km. The 2nd team reported a constant decrease in temperature at a rate of 1.0 °C/hour as recorded by the mobile thermometer. What would be the rate of change (in time) of temperature as measured by any of the volunteers at a fixed location (for instance the one marked by an "X") ? (10%)

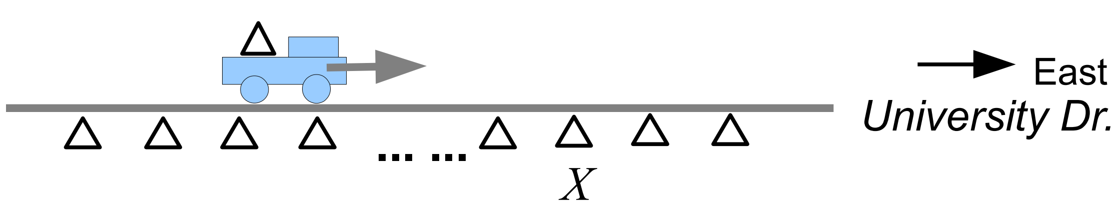

Fig. 2

Additional Note:

For Prob. 1, the expected outcome will be a plot with the flow field and the outline of the "dyed blob" at different times. For example, if the velocity field in Prob. 1 is replaced by a pure deformation field with $u(x, y) = -x$, $v(x, y) = y$, and if we are asked to track the blob to $t = 1$, the final plot will be as shown in Fig. 3. The circle filled in black is the "initial" blob given in Prob. 1. This plot was made using the "quiver" and "fill" commands of Matlab.

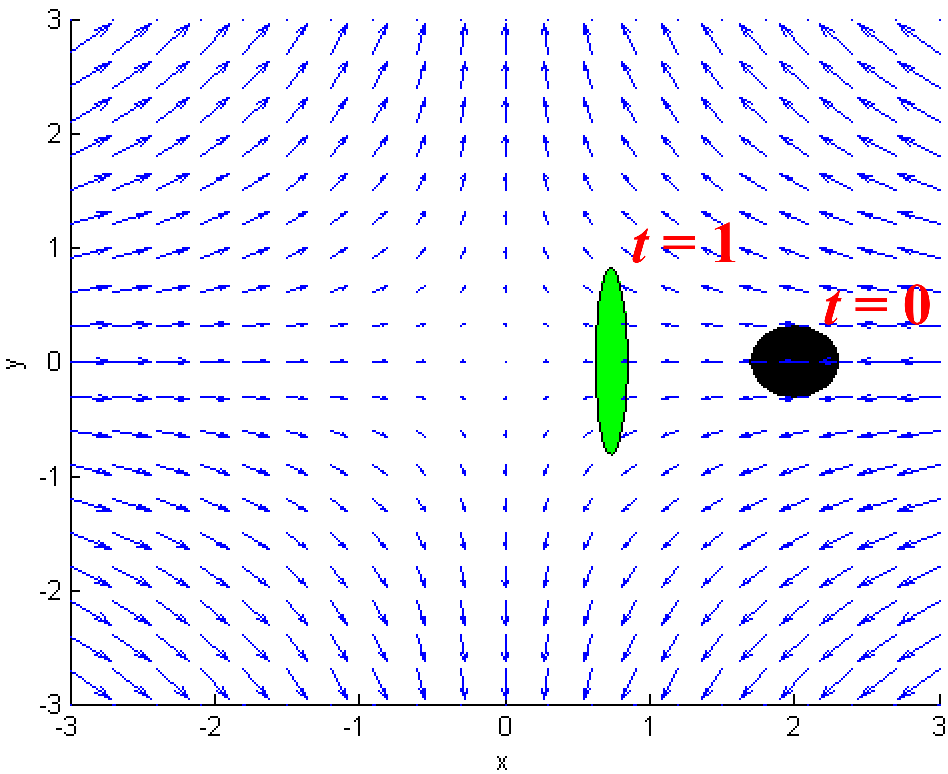

Fig. 3