MAE571, Fall 2014, Homework #4

Due Wednesday, November 12, at the start of class. Must submit the print out of computer code to receive credit.

Prob 1 (100%)

Background: We have discussed the classic similarity solution for a 2-D, steady laminar boundary layer over a flat semi-infinite plate. As a quick summary (see Sec 8.3 for the detailed set-up of the problem), consider the simplified equations for a 2-D steady flow,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} \quad , \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad , \qquad (2)$$

with the boundary conditions (for $x \geq 0$),

$u(x, 0) = 0$,

and

$u(x, y) \rightarrow U$ as $y \rightarrow \infty$.

Here, $x = 0$ is the leading edge of the semi-infinite plate and U is the x-velocity of the impending flow which is uniform for $x < 0$. We seek the solution only over the domain of $x \geq 0$ and $0 \leq y < \infty$. From Eq. (2), defining a stream function (ψ) by $(u, v) \equiv (\partial\psi/\partial y, -\partial\psi/\partial x)$, all terms in Eq. (1) can be expressed as a combination of ψ and its derivatives. Next, we seek a similarity solution of $u(\eta)$ where $\eta \equiv y/g(x)$ and $g(x) = (2\nu x/U)^{1/2}$. After a lengthy derivation (see Sec 8.3), the original PDE and boundary conditions are transformed to

$$f''' + f\,f'' = 0 \, , \qquad (3)$$

with the boundary conditions,

(i) $f(0) = 0$,
(ii) $f'(0) = 0$,
(iii) $f'(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$,

where $(\ )' \equiv d(\ )/d\eta$, $\psi(\eta) = Ug(x)f(\eta)$, and $u(\eta) = U\,f'(\eta)$.

The tasks: Solve Eq. (3) with the boundary conditions (i)-(iii) numerically to obtain $f(\eta)$ and $f'(\eta)$. The shooting method is recommended but you may choose any numerical scheme as you prefer. Use the solution to complete the following:

(a) Plot the profile of $u(\eta)/U$ as a function of η. This is essentially a reproduction of Fig. 8.8.

(b) Plot the contours of $\psi(x, y)$ and $v(x, y)$ in the x-y plane.

(c) If the boundary layer thickness at a given x, $\delta(x)$, is defined as the value of y at which $u(x,y) = 0.9\ U$, make a plot of $\delta(x)$ as a function of x.

(d) Make a plot of selected profiles of $u(x,y)$ as a function of y at different locations of x, in the fashion of Fig. 8.2. It will be particularly illustrative if you can combine the plots for Part (c) and Part (d) into a single figure.

(e) At $x = 1$, plot the profiles of the three terms in Eq. (1) as a function of y. There should be three curves for the three terms. If your solution is good, the sum of the first two (inertial) terms should equal the third (viscous) term. Check whether this is true by also showing the curve of the sum of the first two terms and examining how well it matches the curve for the third term. This figure serves to illustrate the local balance among the inertial and viscous terms.