MAE571, Fall 2016   Homework #2 (13 points)

1. As a combination of the two simple examples discussed in Lecture 7, consider the flow system illustrated in Fig. 1 with a uni-directional flow $u(y)$ bounded by two plates separated by a distance of H. The flow is driven by an imposed, uniform, positive pressure gradient force in the x-direction (i.e., pressure decreases in the positive x-direction). In addition, the top plate is dragged by an external force at a constant velocity in the x-direction. The bottom plate is fixed in space. No-slip boundary conditions for velocity apply to both top and bottom boundaries. At the steady state, the flow is in the x-direction and is independent of x and z. The density (ρ) and kinematic viscosity (ν) of the fluid are constants. With this setup, the governing equation for $u(y)$ is

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad ,$$

where the P. G. F. term is a positive constant, $-\rho^{-1}(\partial p/\partial x) \equiv P > 0$. Find the solutions for the system for the two cases with (i) the top plate moving at a speed of $U > 0$, i.e., it is dragged to the right as shown in Fig. 1; (ii) the top plate moving at a speed of $-U$, i.e., it is dragged to the left. Express the solution of $u(y)$ in terms of the parameters U, P, H, and ν. Giving $\nu = 10^{-6}$ m^2/s, $P = 2\times10^{-6}$ m/s^2, $H = 0.2$ m, and $U = 0.01$ m/s, plot $u(y)$ (in m/s) as a function of y for the two cases (i) and (ii) described above. It is recommended that the two curved be collected in a single plot for a comparison. Briefly discuss how momentum balance is maintained in the interior of the flow at the steady state for the two cases.
(2 points)

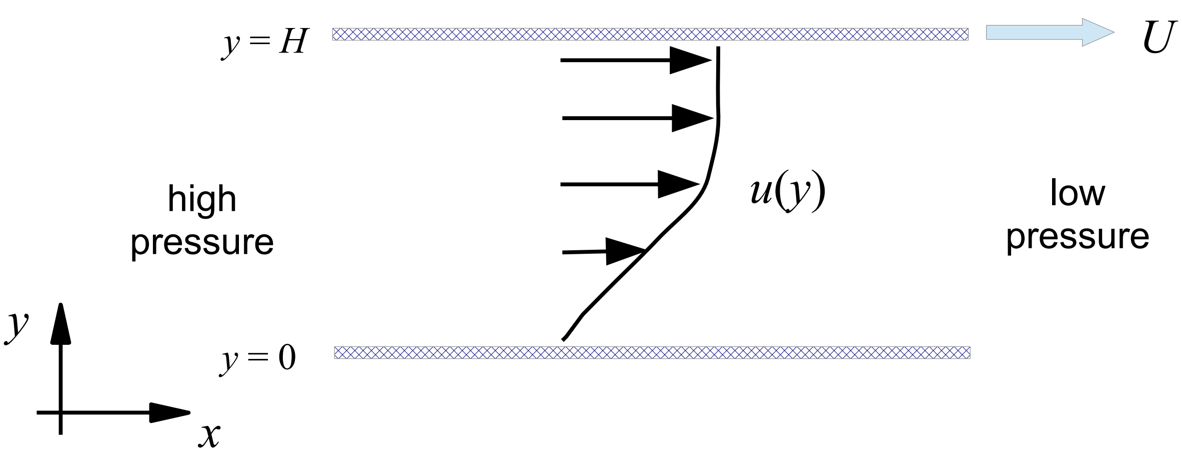

Fig. 1 (The profile shown is arbitrary and not representative of the actual solution.)

2. (a) Solve Problem 2.4 in Acheson's textbook. (b) In addition, find the solution for $u_2(y)$, i.e., the velocity profile for the upper layer. (c) Giving $g = 9.8$ m/s^2 and $\alpha = 9°$ (or 0.05 π radian), plot the vertical profile of u-velocity over the entire depth of the system, i.e., covering both layers, for the two cases: (i) $h_1 = 0.03$ m, $h_2 = 0.02$ m, $\nu_1 = 0.0002$ m^2/s, and $\nu_2 = 0.0001$ m^2/s; (ii) $h_1 = 0.01$ m, $h_2 = 0.04$ m, $\nu_1 = 0.0001$ m^2/s, and $\nu_2 = 0.0003$ m^2/s;
[Note: This problem is a generalization of the example discussed in pp. 38-40 in Acheson's textbook. Please read the textbook for useful background.] (5 points)

3. As discussed in class, in the interior of the flow the viscous term acts to redistribute momentum but does not destroy the total momentum of the fluid system. At the same time, the viscous term does dissipate kinetic energy, generally turning it into heat. Consider an idealized system used in class to isolate the effect of viscosity: A 1-D uni-directional flow, $u(z,t)$, is defined on the unbounded domain of $-\infty < z < \infty$ and governed by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} \quad , \qquad \text{Eq. (1)}$$

where ν is kinematic viscosity. The natural boundary conditions are $u \to 0$ and $\partial u/\partial z \to 0$ as $z \to \pm\infty$. It is assumed that the flow is uniform in the x- and y-direction. Define the total momentum, M, and total kinetic energy, E, as

$$M(t) \equiv \int_{-\infty}^{\infty} \rho u \, dz \quad , \quad E(t) \equiv \int_{-\infty}^{\infty} \left(\frac{1}{2}\rho u^2\right) dz \quad ,$$

we showed in class that M is conserved for the system. On the other hand, E is not conserved. This is known as *viscous dissipation* (or *Rayleigh dissipation*), as discussed in a more general context in Section 6.5 of Acheson's textbook.

(a) If the initial profile of u is given as $u(z,0) = U \exp[-(z/L)^2]$ (a "Gaussian jet" as illustrated in Fig 1), find the solution, $u(z,t)$, that satisfies Eq. (1) and its associated boundary conditions. Show that the solution indeed satisfies $dM/dt = 0$.

(b) Consider the solution from Part (a) with $U = 0.1$ m/s, L = 0.1 m, and assume that the system consists of water with $\rho = 1000$ kg/m^3, $\nu = 10^{-6}$ m^2/s, and specific heat Cp = 4200 J/kg°C. Assuming that the loss of kinetic energy by viscous dissipation that occurs within the 3-D box bounded by -0.2 m $\leq x \leq 0.2$ m, -0.2 m $\leq y \leq 0.2$ m, and -0.2 m $\leq z \leq 0.2$ m, is used to uniformly heat the water within that box, calculate the heating rate, dT/dt, for the water in the box, at $t = 0$ and $t = 10$ minutes. If the temperature in the box is 20°C at $t = 0$, what would be the temperature at $t = 30$ minutes? (This can be calculated by integrating the heating rate, dT/dt, over the 30-min period.) Does viscous dissipation provide significant heating for the system? (5 points for Prob 3)

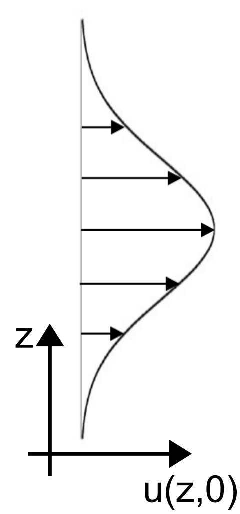

Fig. 2

4. A mini submarine is 10 m long and typically operates at a speed of 0.5 m/s undersea. If one attempts to test a 5:1 scaled-down model of the submarine (i.e., the toy model is 2 m long, with all other dimensions adjusted in proportion) in a wind tunnel, what would be the appropriate wind speed to impose in the tunnel in order for the flow around the toy submarine to emulate the flow around the real submarine moving at its typical speed? Here, we assume that the flow (for either air or water) is incompressible, thermodynamic effects can be neglected, etc., such that "Reynolds number similarity" holds. (Since viscosity generally depends on temperature, you might assume a typical operating temperature of 20°C for air and water in the calculation.) (1 point)