MAE571, Fall 2016 Homework 5+6 (27 points)

Prob 1 (10 points)
Background: In our discussion of the Hele-Shaw flow, it was argued that when $Re(h/L)^2 << 1$, the fluid system behaves somewhat like a 2-D irrotational flow with the stream function governed by Laplace's equation. Here, Re is Reynolds number, h is the vertical separation between two plates, and L is the horizontal scale of the flow. Since the solution of Laplace's equation cannot have a local maximum or minimum in the interior of the domain, it implies that there cannot be a closed streamline or separation of the flow. For example, Case (A) as shown in Fig. 1 (modified from Fig. 7.9 in the textbook) is typical of a Hele-Shaw flow, while Case (B) is not allowed for such a flow. If the condition, $Re(h/L)^2 << 1$, is not satisfied but the flow has a small to intermediate Reynolds number, one may expect a typical laminar flow like Case (B). At a higher Reynolds number, the flow eventually becomes turbulent, close to Case (C).

Task: Use Ansys-Fluent to simulate a flow through a 3-D two-section channel as shown in the right panel of Fig. 1. A uniform velocity, $u = U$, is imposed at the narrow *velocity inlet*. An *outflow* boundary condition is imposed at the wider outlet as shown. A no-slip boundary condition for the velocity is imposed at all solid walls. The flow is incompressible with constant density and viscosity. With all other parameters fixed, when h is very small or when viscosity is very high (such that $Re(h/L)^2 << 1$ is satisfied) the system is expected to approach a Hele-Shaw flow. With a reduction in viscosity or an increase in h, the flow may transition from Hele-Shaw (Case (A)) to a typical viscous flow with an intermediate Reynolds number (Case (B)). Your simulations will focus on the flow regimes of Case (A) and (B), before the flow becomes turbulent. (This will require some adjustments of the inlet velocity, etc.) The following are the key tasks to complete:

(1) Demonstrate a pair of simulations which differ only in h and which show contrasting behaviors of Case (A) (under a smaller h) vs. Case (B) (under a larger h). Clearly describe the setup of the runs, including the values of all of the external parameters (inlet velocity, viscosity, all relevant dimensions of D_1, D_2, h, and the stream-wise length of the two segments of the channel). Is the condition, $Re(h/L)^2 << 1$, satisfied for the simulation that exhibits the "Case A" behavior?

(2) Take the specific simulation in Part (1) that behaves like Case (B) and increase the viscosity (but keep everything else fixed) until the flow transitions to Case (A). Record the value of the viscosity for this case. Is the condition, $Re(h/L)^2 << 1$, satisfied for this case?

(3) In (1) and (2), when a "Case A" type of flow pattern is produced by a simulation, we expect that the condition, $Re(h/L)^2 << 1$, is satisfied. Yet, a question remains whether one should use D_1 or D_2 as the horizontal length scale,"L", in that condition. Note that D_1 is the width of the fluid jet that is being injected into the wider channel, while D_2 is the width of the wider channel itself (or, the largest possible horizontal extent of the flow). Analyze the results of Part (1) and (2) and design/perform additional simulations to address this issue. For example, with a fixed D_2, would it be more difficult to create a "Case A" type of flow with a decreasing D_1? What would be the behavior of the flow (close to Case A or Case B) when $h \approx D_1 << D_2$?

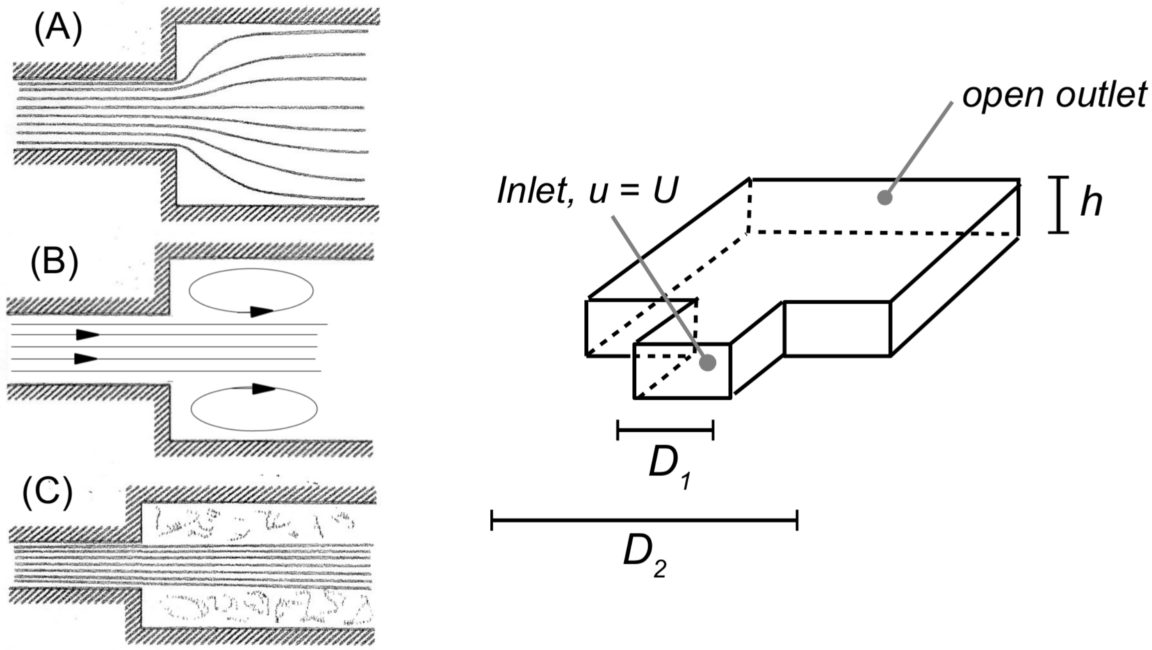


Fig. 1

Prob 2 (10 points)
(a) Based on the discussion in Section 7.9, use the equation for a thin film down a slope (Eq. 7.56) to solve the evolution of the free surface, $h(x,t)$, when the initial shape of $h(x,t)$ is given as

$$
\begin{aligned}
h(x,0) &= 0.2\,(x - 0.2\,x^2)\ \text{cm}, \quad \text{if } 0 \le x \le 5 \text{ cm},\\
&= 0 \qquad\qquad\qquad\quad\ , \quad \text{elsewhere.}
\end{aligned}
$$

See Fig 2 for a sketch of the initial state (not drawn to scale). Moreover, use the parameter setting of $g = 980$ cm/s^2 for gravity, $\nu = 50$ cm^2/s for the kinematic viscosity (which nominally represents honey at room temperature), and $\alpha = 30°$ as the angle of the slope. Plot $h(x,t)$ as a function of x at $t = 0$, 0.5, 1, and 2. It is recommended that all four curves are collected in a single plot.

(b) The equation used in Part (a), Eq. 7.56, was derived with the assumption that the angle α is not too small such that $\sin(\alpha)$ and $\cos(\alpha)$ are of the same order of magnitude. Given that $\partial h/\partial x \sim O(h/L) \ll 1$, this allows us to eliminate the second term in the r.h.s. of Eq. (7.54), which greatly simplifies the final equation for $h(x,t)$. If α is very small such that $\sin(\alpha) \sim O(h/L) \ll 1$ and $\cos(\alpha) \sim O(1)$, the two terms in the r.h.s. of Eq. (7.54) will be comparable in magnitude such that both must be kept in the equation. In this case, try to keep the full Eq. (7.54) and complete the remaining steps in the derivation to obtain the governing equation for $h(x,t)$ (as a generalization of Eq. 7.56) which is valid for any value of α. Discuss whether the method (analytic or numerical) that you used in Part (a) can be used to solve the generalized equation.

Note for Part (a):
(i) According to Eq. (7.56) and related discussions in the textbook, if initially $h = 0$ at a certain location, that point will not propagate in the x-direction at all. This means that for the initial condition given in Part (a), the point at $x = 0$ and $x = 5$ cm will not move. Thus, for this problem, the fluid "blob" will deform within $0 \le x \le 5$ cm but the whole blob will not just drift down the slop. However, the "center

of mass" of the blob will move down the slope.
(ii) The problem can be solved either analytically by the Method of Characteristics, or by a straightforward numerical (e.g., finite difference) method. The former requires solving a quartic equation which could be done either analytically or numerically.

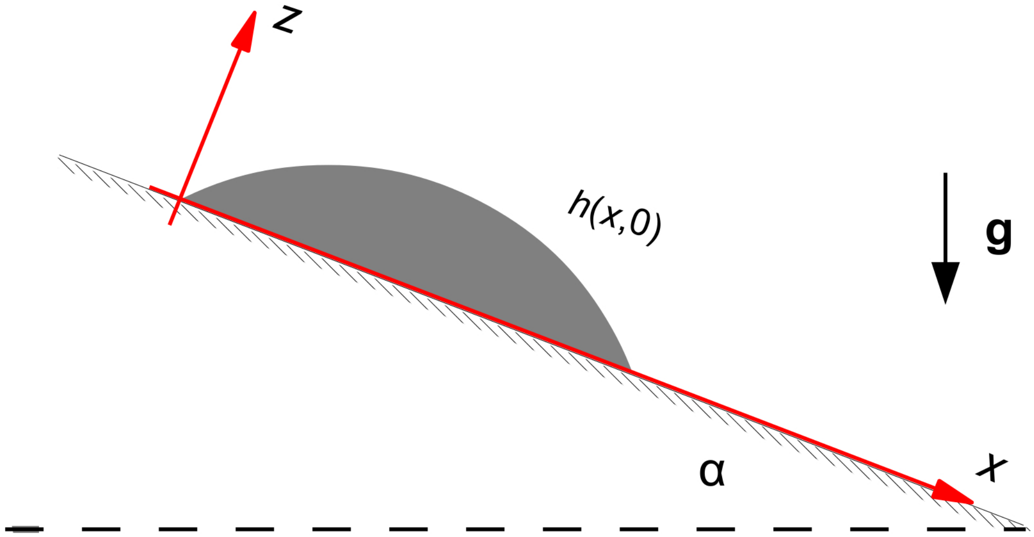
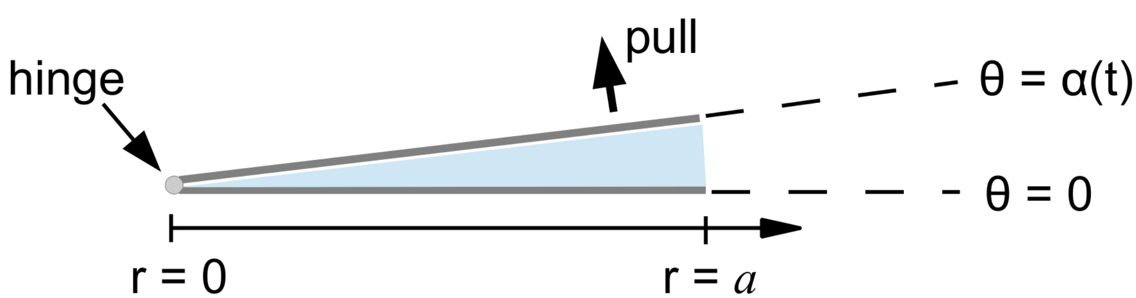


Fig. 2

Choose <u>one of the following three problems</u> to solve (7 points)

Prob 3A. Complete the exercise in Prob. 7.9 in the textbook. The following diagram illustrates the set up of the system.



Prob 3B. Complete the exercise in Prob 6.13 (proof of Reynolds Transport Theorem) in the textbook. Reynolds Transport Theorem is given in p. 206.

Prob 3C. Complete the exercise in Prob 8.5 in the textbook. Note that the sketch of velocity profile should be for the total u-velocity, i.e., $u = U + u_1$, instead of just u_1 .