MAE578 Fall 2011 Homework #2

Wien's displacement law

1. Given Planck function $B_\lambda(T)$ in Eq. (A-1), for a fixed temperature the peak wavelength of the radiative energy spectrum is located where $\partial B_\lambda(T)/\partial\lambda = 0$. Show that this leads to *Wien's displacement law* which states that $\lambda_{peak} = \alpha/T$, where λ_{peak} is the peak wavelength, α is a constant (please determine its value), and T is temperature in °K. (Since λ_{peak} decreases with an increasing temperature, a star that looks blue is hotter than one that looks red.) What are the values of λ_{peak}, in µm, for T = 6000 °K and T = 270 °K? **(0.5 point)**

Radiative equilibrium

2. A hypothetical structure, called a "Dyson sphere", is a structure that an advanced civilization might build to harvest as much energy as possible from the star of their "solar system". To explain the concept, note that for our solar system most of the radiative energy emitted by the Sun escapes to the vast universe, with only an extremely small fraction of it being intercepted by the Earth for the use of human beings. As envisioned by F. Dyson*, to maximally harvest solar energy, one might build a spherical shell, for instance with a radius of 1 A.U. and centered at the Sun, that completely encloses the Sun. Such a structure, if it exists, would have a much lower temperature than the Sun itself such that the entire solar system would appear (to an observer outside the solar system) to emit infrared radiation. Dyson suggested a search of "infrared stars" as a way to detect extraterrestrial intelligent life.

Suppose that such a structure with a radius of 1 A.U. is built for our solar system. Moreover, assume that the spherical shell is very thin and highly conductive such that its inner surface (facing the Sun) and outer surface (facing the universe for aliens to observe) have the same temperature. The material used to build the shell is completely absorbing of solar radiation (i.e., its albedo is zero.) **(a)** What would be the temperature, in °K, of the spherical shell at radiative equilibrium? **(b)** Using Wien's displacement law, what would be the peak wavelength, in µm, of the radiation emitted by this Dyson sphere? **(c)** What would be the surface temperature of the Dyson sphere if its radius is 2 A.U. instead of 1 A.U.? **(3 points)**

* The original article is F. J. Dyson, 1960, *Science*, Vol. 131, pp. 1667-1668. A copy of it can be found at this website: http://www.islandone.org/LEOBiblio/SETI1.HTM. You do not need to read the article to solve this problem.

Radiative heating and vertical temperature profile

3. **(a)** Work out Prob 5 of Chap.2 in M&P textbook. Note that since all atmospheric layers are "completely absorbing of IR radiation", the "planetary emission temperature" T_e is simply the temperature of the top layer, T_1. **(1 point)** **(b)** Adopting a 5-layer model (N = 5 in Fig. 2.12) with $S_o/4 = 342\ W\ m^{-2}$ and surface albedo $\alpha = 0.1$, calculate the temperatures for all layers, T_n, n = 1, 2, ..., 5, along with the surface temperature, T_S. Plot the vertical temperature profile of the atmosphere using these 6 data points. Comment on the results. **(0.5 point)**

4. Repeat Part (b) of Prob 3 but assume that the atmospheric layers have an IR absorptivity of 0.9 (instead of 1.0). That is, 10% of the IR radiation can now transmit through an atmospheric layer. (It may be convenient to denote the "transmissivity" as $\beta = 0.1$ in your mathematical derivation.) Repeat the calculation for the 5-layer model to obtain T_n for all layers and the surface temperature, T_S. Plot the temperature profile. Compare it with the case of $\beta = 0$ in Prob. 3b by plotting the difference in the temperature profile between the two cases. Comment on the results. **(3 points)**