MAE578 Fall 2011 Homework #3

Hydrostatic balance

1. An idealized temperature profile for the atmosphere is given as

$$T(z) = T(0) - \alpha z \;, \qquad 0 < z \leq 10 \text{ km}$$
$$T(z) = T(10 \text{ km}) + \beta\,(z - 10 \text{ km}) \;, \qquad 10 \text{ km} < z \leq 30 \text{ km} \;,$$

with both α and β positive. This would nominally represent the decrease of temperature with height in the troposphere and increase of temperature with height in the stratosphere. See sketch in Fig. 1.

(a) Assume that the atmosphere is in hydrostatic balance and consider α = 6 °K/km, β = 3 °K/km. Given the surface temperature T(0) = 300°K and surface pressure p(0) = 1000 hPa (1 hPa = 100 Pascal), solve for T(z), p(z), and density $\rho(z)$ and plot all three profiles from the surface to 30 km. You may keep g (gravitational acceleration) and R (ideal gas constant) as constants.

(b) Repeat (a) but consider α = 11 °K/km. Compare the vertical profiles of this case with their counterparts in (a).

(c) For the case in (a), what is the thickness (in meters) of the atmosphere between the 900 hPa and 500 hPa levels? What is the pressure at z = 30 km?

(d) In general (for any given α and β), if we keep the formula for T(z) intact and fix p(0) but decrease T(0) by 1°K (effectively shifting the temperature profile by a constant 1 °K - see gray lines in Fig. 1), will the pressure at z = 30 km increase or decrease? Interpret your result physically. **(4.5 points)**

Potential temperature and static stability

2. Using the temperature and pressure profiles in Prob 1, deduce the potential temperature profile, $\theta(z)$, and the profile of $d\theta/dz$, for the "troposphere" ($0 < z \leq 10$ km). Plot these two profiles for the cases with α = 6 °K/km and α = 11 °K/km from Prob 1 and discuss their static stability. **(2.5 points)**

Buoyancy

3. A big tank is filled with a thick layer of cooking oil (density ρ = 800 kg/m^3) on top of a layer of water (ρ = 1000 kg/m^3) that is 1 m deep, as shown in Fig. 2. A small plastic ball with ρ = 950 kg/m^3 is released from the bottom of the tank. Ignoring the effects of friction and surface tension (in reality these effects are significant), how far upward will the ball penetrate into the oil layer before it stops? **(1 point)**

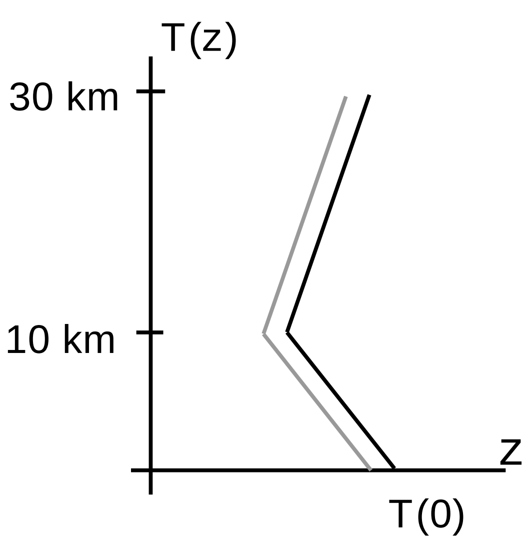

Fig. 1

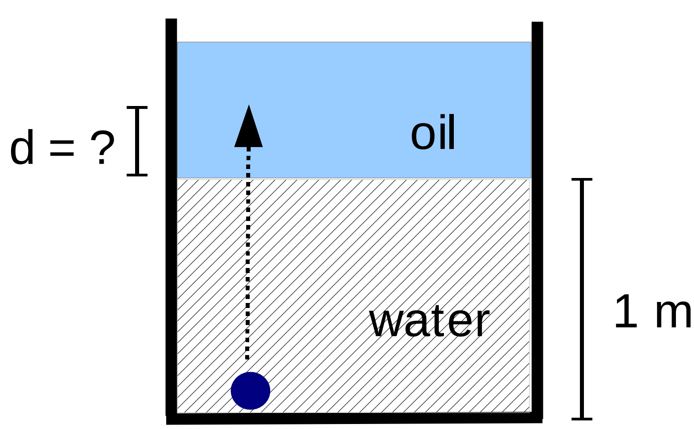

Fig. 2