Problem 1

(a) At $p = 1000$ hPa, $T = 28^\circ C = 301.16^\circ K$

$$E_s(T) = 611 \text{ Pa} \cdot e^{0.067 \times 28} \approx 39.88 \text{ hPa}$$

$$q_s \approx \frac{E E_s}{p} = 0.62 \cdot \frac{39.88}{1000} \approx 0.0247$$

$$RH = \frac{q}{q_s} = \frac{0.013}{0.0247} \approx 52.6\%$$

$$\frac{E}{E_s} \sim RH = 0.526 \Rightarrow E = 0.526 \times 39.88 = 20.97 \text{ hPa}$$

(b) Assume that the air parcel does not reach saturation level at 920 hPa. (This assumption will be verified later). $\Rightarrow q$ is conserved.

Adiabatic process $\Rightarrow \theta$ is conserved

$$T_{1000} = \theta_{1000} = \theta_{920} = T_{920} \left( \frac{1000 \text{ hPa}}{920 \text{ hPa}} \right)^{\gamma/p} \Rightarrow T_{920} = 294.1^\circ K$$

$\Rightarrow T_{920} = 20.9^\circ C$

$\Rightarrow E_s = 611 \text{ Pa} \cdot e^{0.067 \times 20.9} \approx 24.83 \text{ hPa}$

$$q_s \approx \frac{E E_s}{p} = 0.62 \times \frac{24.83 \text{ hPa}}{920 \text{ hPa}} \approx 0.0167 > 0.013$$

(This verifies the original assumption)

$\Rightarrow$ The parcel is not saturated at 920 hPa: No condensation.

$$RH = \frac{0.013}{0.0167} \approx 77.6\%$$
Use hydrostatic eq. and assume dry atmosphere to obtain $p(z)$:
$$p(z) = \left( \frac{T(0) - z}{T(0)} \right) \frac{\rho}{\rho_0} \tag{1}$$

$$\gamma(z) = \frac{\rho(z)}{\rho_{0}} \approx \frac{\rho(z)}{\rho_{0} \left( E \cdot E_{s}(T(z)) \right) / p(z)}$$

$$\Rightarrow \rho(z) = \gamma(z) \cdot E \cdot E_{s}(T(z)) / p(z)$$
$$= 0.9 \cdot e^{-\frac{z}{T(0)} (0.62)} \cdot (0.11) \cdot \frac{0.067 (26.84 - 9z)}{1000 \left[ \frac{300 - 9z}{300} \right] 9.8/287 \times 0.009}, \quad \tag{2}$$

where $z$ is in km.

Using Eq. (1) and the given $T(z)$, we can calculate
$$\Theta(z) = T(z) \left( \frac{1000 \text{ hPa}}{p(z)} \right) \frac{\rho(z)}{\rho_0}$$

and, using Eq. (2),
$$\Theta_{e}(z) = \Theta(z) \cdot \exp \left( \frac{L \cdot \rho(z)}{\rho_0 T(z)} \right).$$

(As a first approximation, we may assume $L \approx \text{const}$)

$\theta/dz$ and $\theta_{e}/dz$ can be readily evaluated by finite difference (or analytically).

See attached plots.

Note that the $\Theta$ profile is stable ($d\Theta/dz > 0$) while $\Theta_{e}$ profile can be unstable ($d\Theta_{e}/dz < 0$) in the lower troposphere due to moisture effect.
Plots for prob 2. (Thanks to John Westerdale)
Problem 3

(a) Neutral stability \[ \Rightarrow \frac{d\Theta}{dz} = 0 \]
\[ \Rightarrow \Theta_{sfc} = \Theta_{tropopause} \]
\[ \Rightarrow T(0) = T(tropopause) \left( \frac{1000 \text{ hPa}}{P_{tropopause}} \right)^{\frac{R}{P}} \]
\[ \Rightarrow P_{tropopause} = 1000 \text{ hPa} \times \left( \frac{305.16}{193.16} \right)^{-\frac{R}{P}} \approx 201.7 \text{ hPa} \]

(b) \[ \Theta = \text{const} = T(0) \]
\[ \Rightarrow T(z) = T(0) \left[ \frac{P(z)}{P(0)} \right]^{-\frac{R}{P}} \left[ \frac{P(z)}{P(0)} \right]^{\frac{R}{P}} \]

By hydrostatic eq., we have
\[ \frac{dp}{dz} = -\left( \frac{3}{RC} \right) P^{1-\frac{R}{P}}, \text{ where } C = T(0) \left[ \frac{P(z)}{P(0)} \right]^{-\frac{R}{P}} \]
\[ \Rightarrow \int_{0}^{z} P^{\frac{R}{P} - 1} dp = -\frac{3}{RC} \int_{0}^{z} dz = -\frac{3}{RC} z \]
\[ \Rightarrow P(z) = P(0) \left[ 1 - \frac{3}{T(0)} \frac{dz}{P(z)} \right]^{\frac{R}{P}} \text{ where } \Gamma_{d} = \frac{3}{P(0)} \]