MAE578 Fall 2011 Homework #7

Thermal wind relation

1. Solve Prob 9 of Chap. 7. Note that "westerly" means *eastward*. Please assume that Fig. 7.29 depicts the Southern Hemisphere (so "Winter Pole" means the South Pole). Be careful about the sign of the Coriolis parameter, *f*. **(2 points)**

Geostrophic/thermal wind balance and the mean climate state

2. (a) Solve Prob 8 of Chap. 7. **(b)** Compare your estimates of the "pressure surface slope" and *u*-velocity (at 45°N) at 200 mb with Figs. 5.13 and 5.20. **(3 points)**

Absolute vorticity

3. This is a generalization of HW6-Prob 1. **(a)** Ignoring friction, if vertical velocity vanishes (w = 0) for a certain fluid flow, the horizontal components of the momentum equation in the rotating frame can be written as

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial x} + f v \quad , \tag{1}$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y} - f u \quad , \tag{2}$$

where $f = 2\Omega \sin \phi$ is the Coriolis parameter. If the density of the fluid is uniform in the horizontal direction, show that Eqs. (1) and (2) lead to

$$\frac{d(\zeta + f)}{dt} = -(\zeta + f) D \quad , \tag{3}$$

where $\zeta \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the *vorticity* and $D \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is the *divergence* of this flow. Therefore, if $D \equiv 0$ (flow is "non-divergent") we have *conservation of* ($\zeta + f$) *following the motion of a fluid parcel*. The quantity, $\eta \equiv \zeta + f$, is the *absolute vorticity*. Here, we see that the Coriolis parameter is the "vorticity" due to planetary rotation; In literature, *f* is sometimes called the "planetary vorticity". [As noted in HW6, when w = 0 and ρ = constant, by continuity equation D is guaranteed to be zero unless there is a mass source or sink.] **(2 points)**

(b) When $D \neq 0$, Eq. (3) indicates that convergence ($D < 0$) leads to an amplification of absolute vorticity. One can appreciate this by momentarily holding D as a constant, which leads to $\eta(t) = \eta(0)\exp(-D\,t)$. Use it to explain the outcome of "Perrot's experiment" in Sec. 6.6.6 (p. 103). [Hint: The experiment would work only if the water in the tank is standing still when one pulls the plug from below. Otherwise, the vortex that forms at the sinkhole can have either clockwise or counterclockwise rotation, which is what we observe in daily life with a bathtub or kitchen sink. Interested students can read Prob 6 of Chapter 6 for more background.] **(1 point)**

Effect of molecular viscosity

4. Solve Prob 1 of Chap. 6. **(1 point)**

Foucault pendulum

5. A plaque that introduces the Foucault pendulum in PSF Building states that the vertical plane of swing of the pendulum is rotating clockwise with a period of 43 hours and 34 minutes. Try to explain where this number comes from. The plaque, pictured, also provides other pieces of information such as the latitude of Tempe, length of the cable, mass of the ball, etc. They may or may not be relevant but are included here for your reference. **(1 point)**