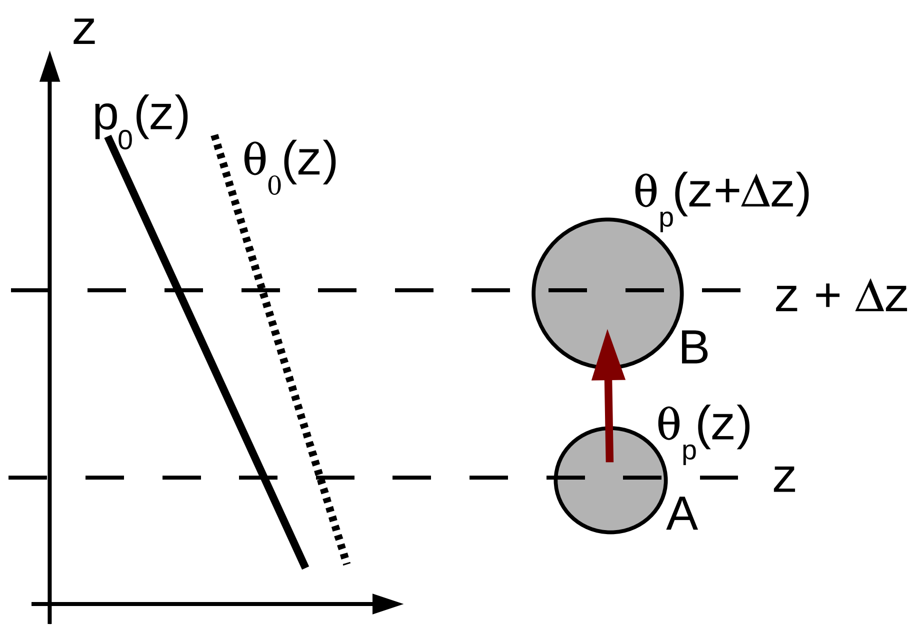
Potential temperature and static stability

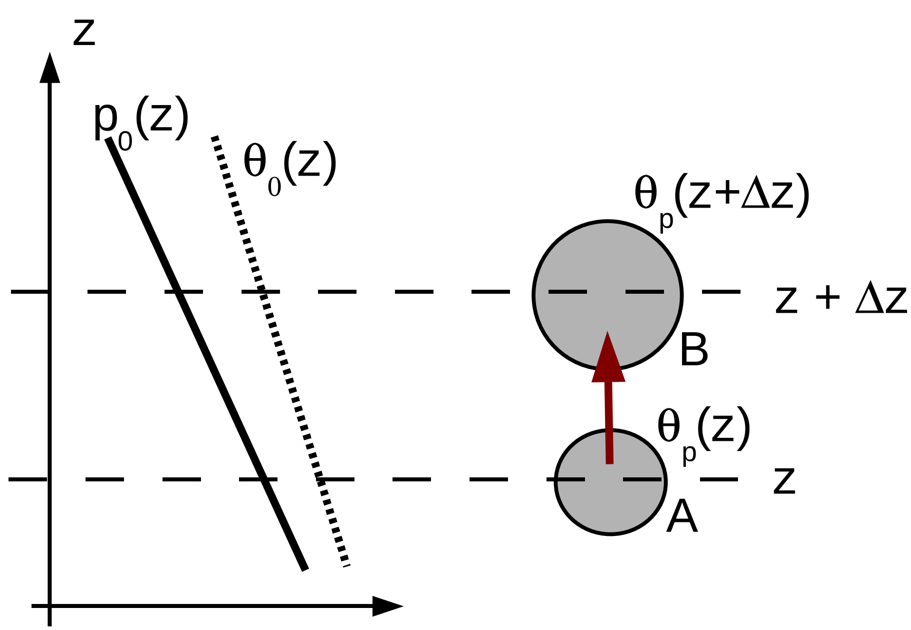
The atmosphere

Stratified in the vertical; $T(z)$, $p(z)$, $\rho(z)$;
Pressure decreases with height (hydrostatic)
Ideal gas: $\rho = p/RT$

As an air parcel is adiabatically lifted (from A to B) to a new height, all of its (ρ, p,T) change, but the potential temperature of the parcel is conserved; $\theta_{p,B} = \theta_{p,A}$ (we will use the subscript "p" to denote "parcel"), or, for the convenience of later discussion, $\theta_p(z+\Delta z) = \theta_p(z)$, where the z and Δz hare indicate the vertical potision of the parcel.



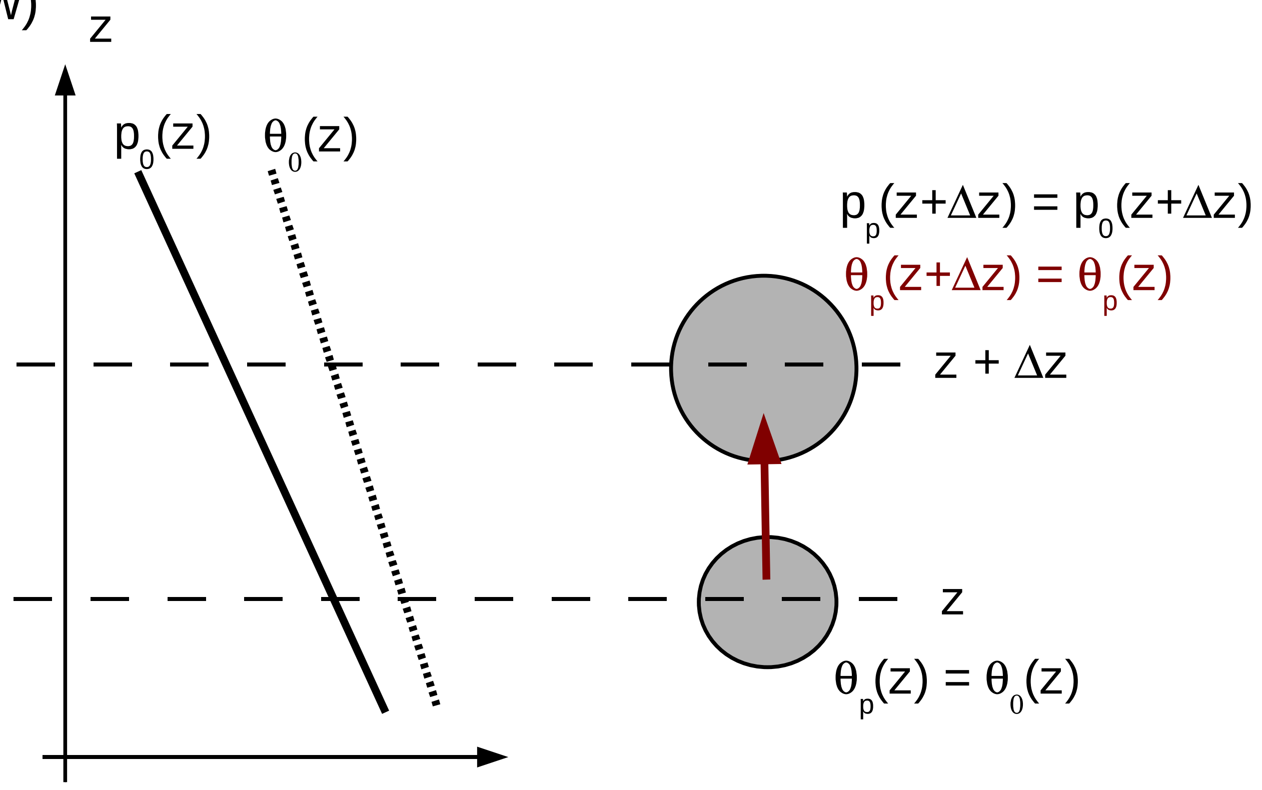
Our ultimate task, as far as static stability is concerned, is to determine whether the lifted parcel at z+Δz is heavier or lighter than its environment at that level. If the parcel is lighter than the environment (density of parcel < density of the environment), then it would experience positive buoyancy and continue rising. In other words, a small upward perturbation of the parcel leads to further upward acceleration of the parcel ⇒ instability.



We will hereafter use the subscript "0" to denote an environmental variable and "p" to denote the value adhered to the parcel

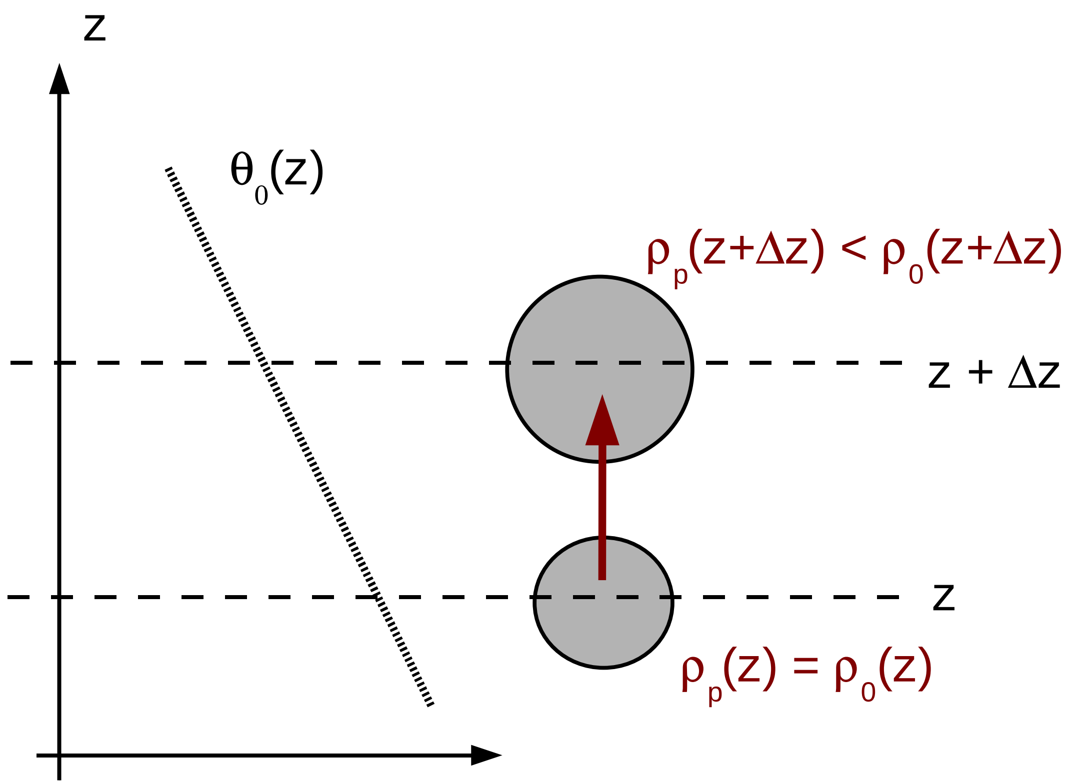
Consider the case when the environmental potential temperature decreases with height, or **$d\theta_0/dz < 0$** (see figure)

1. We carve the parcel out of the environment at z, so $\theta_p(z) = \theta_0(z)$
2. The pressure of the parcel adjusts to its environment, $p_p(z+\Delta z) = p_0(z+\Delta z)$
3. Adiabatic process $\Rightarrow$ θ is conserved for the parcel, so $\theta_p(z+\Delta z) = \theta_p(z)$
4. (1) & (3) $\Rightarrow$ $\theta_p(z+\Delta z) = \theta_0(z) > \theta_0(z+\Delta z)$
5. Since $p_p(z+\Delta z) = p_0(z+\Delta z)$, $\theta_p(z+\Delta z) > \theta_0(z+\Delta z)$ implies $T_p(z+\Delta z) > T_0(z+\Delta z)$ (just look up the definition of θ)
6. Since $p_p(z+\Delta z) = p_0(z+\Delta z)$, $T_p(z+\Delta z) > T_0(z+\Delta z)$ implies $\rho_p(z+\Delta z) < \rho_0(z+\Delta z)$ (by invoking ideal gas law)



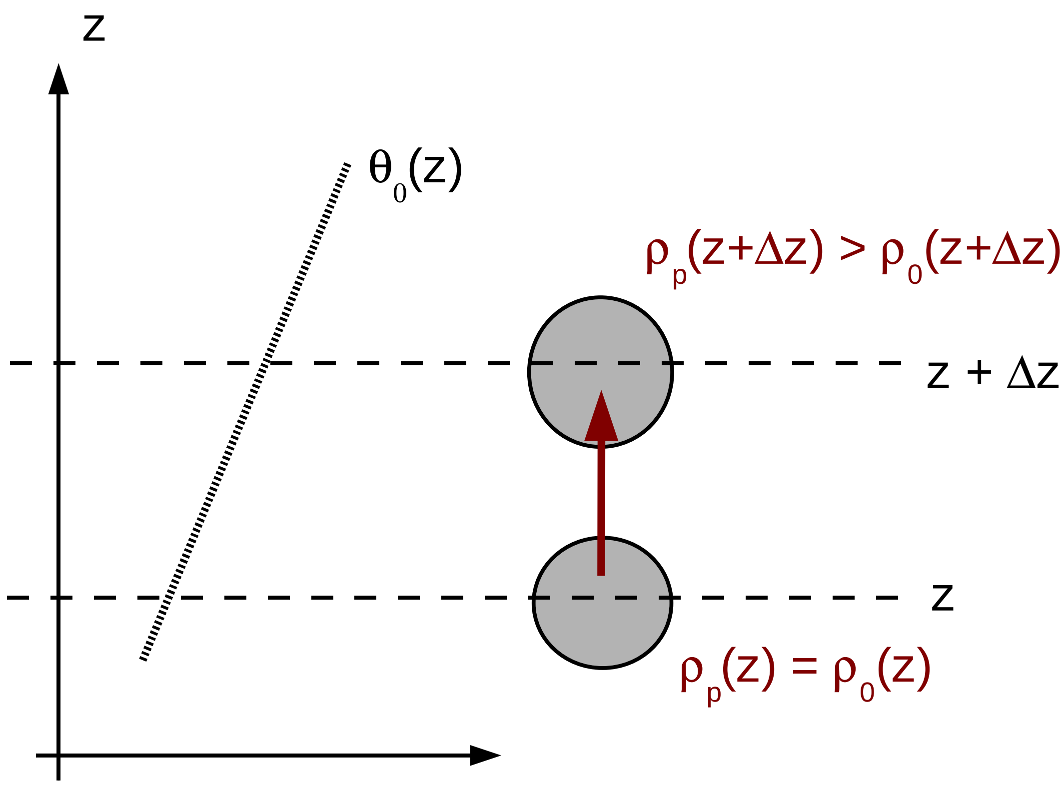
Consider the case when the environmental potential temperature decreases with height, or **$d\theta_0/dz < 0$** (see figure)

Conclusion: $\rho_p(z+\Delta z) < \rho_0(z+\Delta z)$; A parcel that's been lifted adiabatically from z to $z+\Delta z$ is lighter than the density of the environment at $z+\Delta z$. It will continue accelerating upward $\Rightarrow$ The case with **$d\theta_0/dz < 0$** is **statically unstable**



If one repeats the analysis by considering an environment with **$d\theta_0/dz > 0$** (see figure), it can be shown that $\rho_p(z+\Delta z) > \rho_0(z+\Delta z)$; A parcel that's been lifted adiabatically from z to z+Δz is heavier than the density of the environment at z+Δz. It will fall back to its original position

$\Rightarrow$ The case with **$d\theta_0/dz > 0$** is **statically stable**



In the preceding discussion, we can also start by pushing the air parcel down instead of lifting it up. The conclusion concerning static stability will remain the same