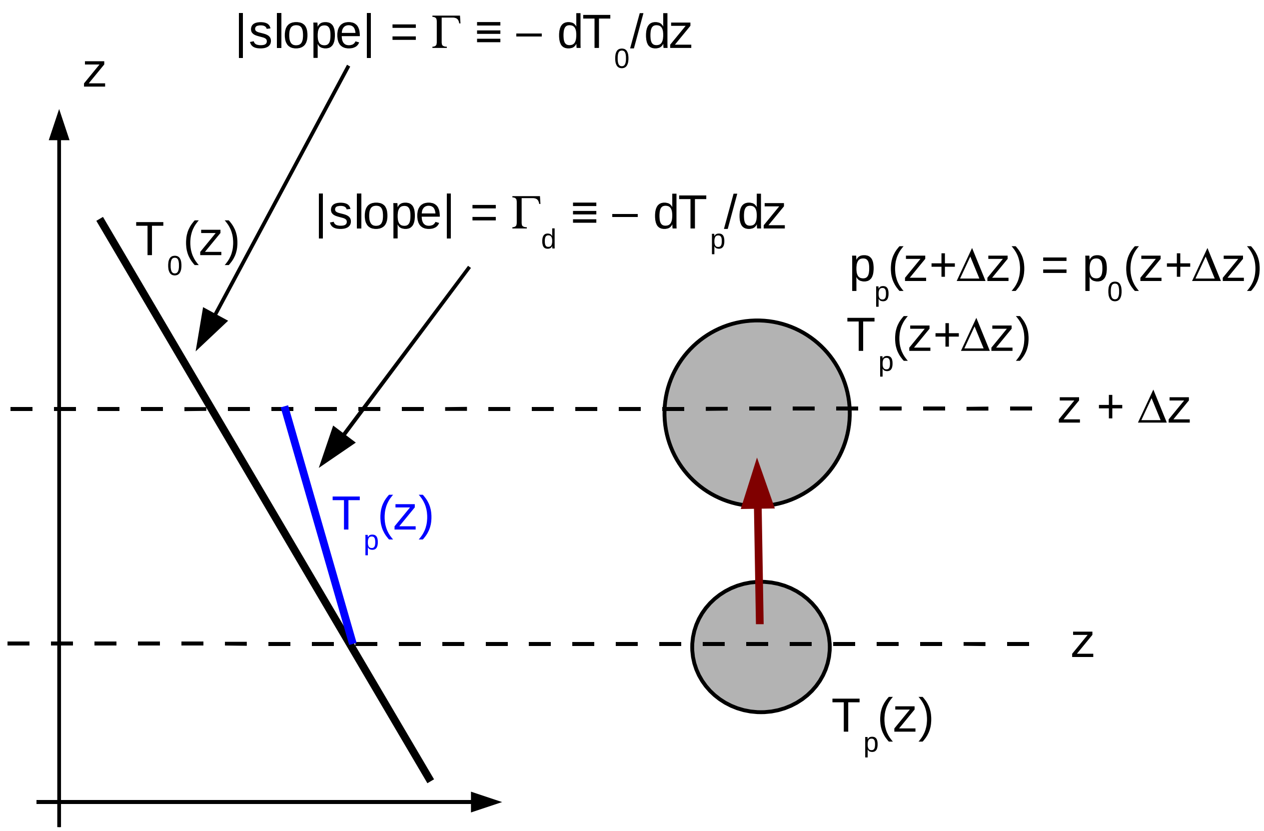
Adiabatic lapse rate and static stability

Let's now consider static stability in terms of temperature profile, $T_0(z)$

Recall from our previous discussion that the pressure of the parcel always adjusts to the environmental value; $p_0(z+\Delta z) = p_p(z+\Delta z) = p^*$ (p^* is a certain constant). By ideal gas law, if $T_p(z+\Delta z) > T_0(z+\Delta z)$ then $\rho_p(z+\Delta z) < \rho_0(z+\Delta z)$, the lifted parcel is lighter than its environment and will continue rising.



To determine stability, it suffices to compare the "environmental lapse rate" for temperature

$$\Gamma \equiv -\, dT_0/dz \,,$$

with the "**adiabatic lapse rate**" for the parcel that undergoes an adiabatic ascent or descent

$$\Gamma_d \equiv -\, dT_p/dz \,.$$

If $\Gamma > \Gamma_d$ (environmental temperature decreases upward more rapidly than the rate of adiabatic cooling for the parcel following an adiabatic ascent), a parcel that's been lifted to $z+\Delta z$ will be warmer, and lighter, than its environment

$\Rightarrow$ The ascent will continue to a greater height $\Rightarrow$ Instability

We now have an alternative form of the criterion for static stability:

$\Gamma < \Gamma_d$: Stable

$\Gamma > \Gamma_d$: Unstable

We will soon demonstrate that this criterion is identical to the criterion based on $d\theta_0/dz$ that we derived before.

Determine the adiabatic lapse rate, Γ_d

As usual, we use the subscript "p" to denote "parcel'. The d/dz in this page is understood as the rate of change following the ascent/descent of the parcel.

Since potential temperature is conserved for the parcel following an adiabatic ascent/descent, we have

$$d\theta_p/dz = 0 \ .$$

From the definition of potential temperature, we also have,

$$\begin{aligned} d\theta_p/dz &= d/dz \{T_p \, (p_S/p_p)^{R/Cp}\} \\ &= (p_S/p_p)^{R/Cp} \, (dT_p/dz + g/C_P) \\ &= (\theta_p/T_p) \, (dT_p/dz + g/C_P) \ . \end{aligned} \qquad \text{Eq. (1)}$$

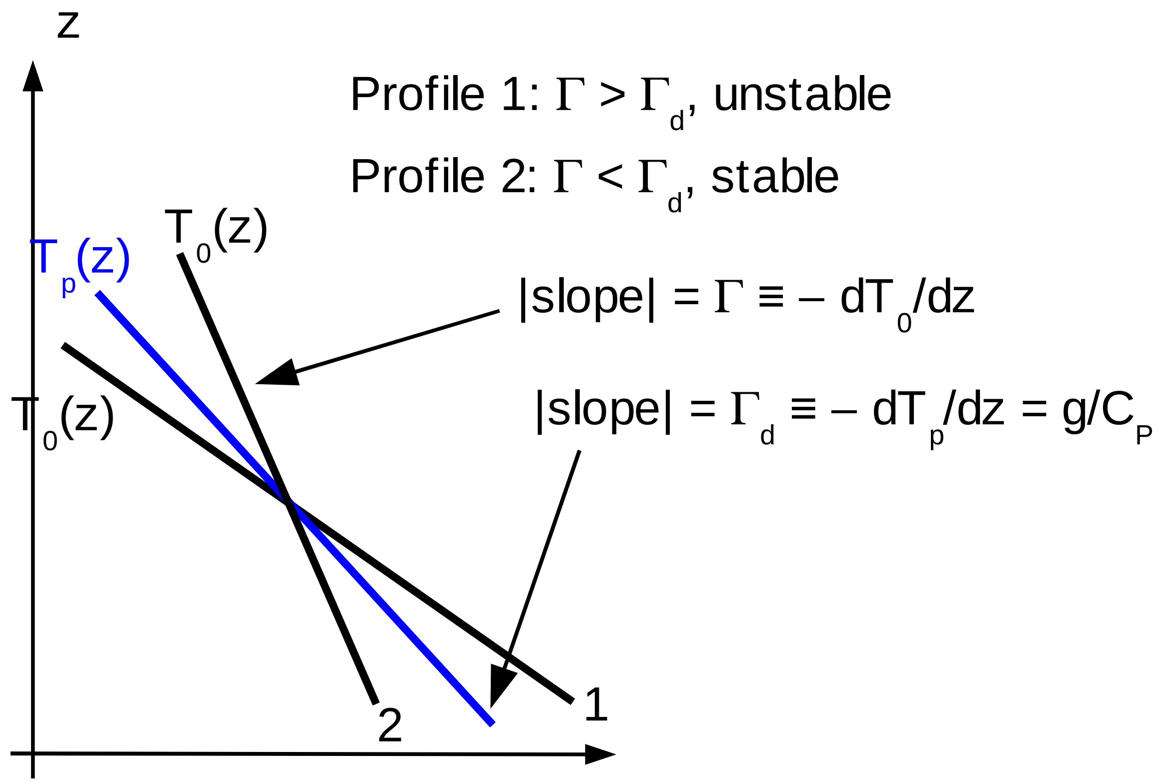
Therefore, from $d\theta_p/dz = 0$ we have $dT_p/dz = -\,g/C_P$, or

$$\Gamma_d \equiv -\,dT_p/dz = g/C_P \approx 10\ °K/km$$

For the environment, we can likewise derive an expression of $d\theta_0/dz$ in terms of Γ and Γ_d:

$$
\begin{aligned}
d\theta_0/dz &= d/dz \{T_0 (p_S/p_0)^{R/Cp}\} \\
&= (\theta_0/T_0)\,(dT_0/dz + g/C_P) \\
&= (\theta_0/T_0)\,(dT_0/dz - dT_p/dz) \\
&= (\theta_0/T_0)\,(\Gamma_d - \Gamma)\ . \qquad \text{Eq.(2)}
\end{aligned}
$$

Since T_0 and θ_0 are always positive, we have now established that the stability criterion, $d\theta_0/dz > 0$, is equivalent to $\Gamma_d - \Gamma > 0$.


z
Profile 1: Γ > Γd, unstable
Profile 2: Γ < Γd, stable
T0(z)
Tp(z)
|slope| = Γ ≡ – dT0/dz
T0(z)
|slope| = Γd ≡ – dTp/dz = g/CP
1
2

We may rewrite Eq. (2) as

$$d \ln \theta_0/dz = (\Gamma_d - \Gamma)/T_0 \ ,$$

or

$$N^2 \equiv g\, d \ln \theta_0/dz = g(\Gamma_d - \Gamma)/T_0 \ .$$

Therefore, for the stable case ($\Gamma_d - \Gamma > 0$), the buoyancy frequency can be determined by the environmental temperature profile as

$$N = \{g(\Gamma_d - \Gamma)/T_0\}^{1/2} \ .$$