MAE 578, Spring 2015   Homework #1

Collaboration is not allowed for this assignment.

Prob 1. (60%)

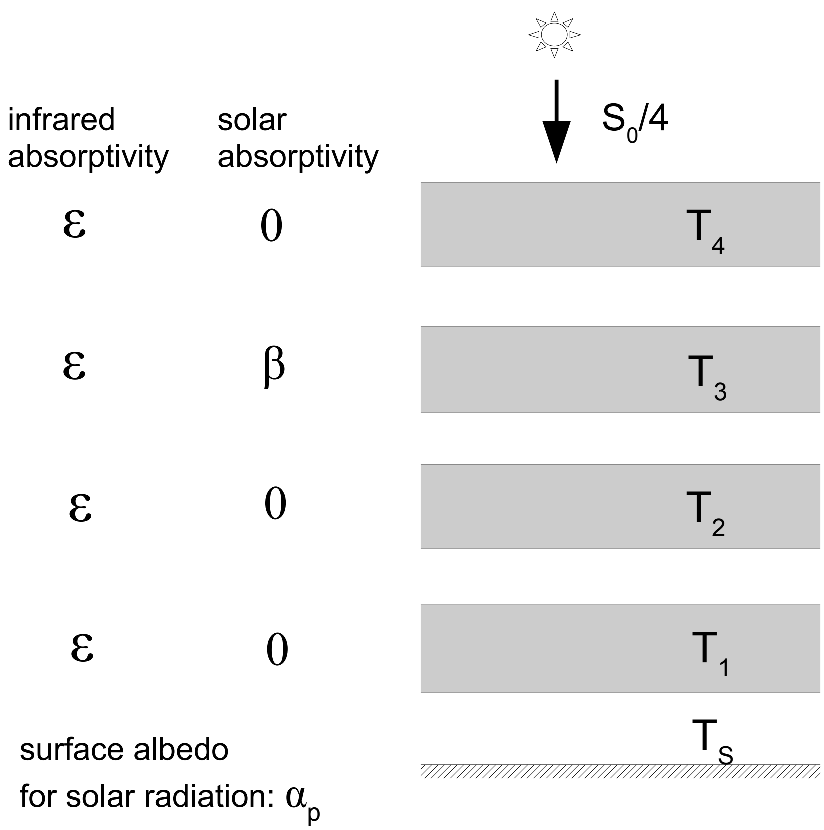
Consider a four-layer model of the atmosphere as illustrated below. All layers are transparent to solar radiation except layer 3 (nominally the "ozone layer"), which has a solar absorptivity of β. All layers have an infrared absorptivity of ε. The surface albedo for solar radiation is α_p. The effective solar input from the top of the atmosphere is $S_0/4$, where $S_0 = 1368$ W m^{-2} is the solar constant. Write a code for the calculation of the temperature profile, $(T_s, T_1, T_2, T_3, T_4)$, for any given parameter values of $(\beta, \varepsilon, \alpha_p)$.

(a) Calculate and plot the temperature profiles for the two cases with $(\beta, \varepsilon, \alpha_p) = (0.3, 0.2, 0.1)$ and $(0.6, 0.2, 0.1)$.

(b) Taking the case of $(\beta, \varepsilon, \alpha_p) = (0.3, 0.2, 0.1)$ as the reference and gradually increase α_p. Show how the temperature of the "ozone layer" (T_3) changes with α_p. Interpret the result.

(c) Taking the case of $(\beta, \varepsilon, \alpha_p) = (0.3, 0.2, 0.1)$ as the reference and gradually increase ε. Show that the local maximum of temperature at layer 3 disappears at large ε. Interpret the result.

(Note: In the special case of $\beta = 0$ and $\varepsilon = 1$ for all layers, a simple analytic solution exists for the system with an arbitrary number of layers. See Prob 5 in Chapter 2 of M&P textbook for the solution and use it to check the integrity of your code.)



Prob 2. (20%)

(a) Adopting the same procedure and definitions used to derive the Solar constant (S_0 = 1368 W m^{-2}), try to estimate its lunar counterpart, the "Lunar constant" L_0, that represents the radiative energy (per unit cross-sectional area) of moonlight intercepted by the Earth. The surface albedo of the Moon is about 0.1. The radius of the Moon is 1700 km and the Moon-to-Earth distance is 380,000 km. Compare L_0 to S_0. In practice, all computer models for weather and climate prediction ignore the effect of moonlight on Earth's radiative energy balance. Is it justified?

(b) During night time, how does the intensity of moonlight that reaches Earth's surface compare to that of the infrared radiation emitted by Earth's surface? You may use T_S = 250°K as the averaged surface temperature for the Earth at night. In this context, does moonlight play a significant role in the radiative energy budget at Earth's surface during the night?

To simplify the calculation, you may adopt these assumptions: (i) Ignore the infrared radiation emitted by the Moon but solely consider the effect of the second-hand sunlight reflected by the Moon that reaches the Earth. (ii) While the Moon-to-Sun distance varies with time, as an approximation we set it to 1 A.U. (iii) Assume that it's full moon all the time. This will lead to an overestimate of L_0, but we are happy to have an order-of-magnitude estimate. (iv) If the spherical geometry of the Moon's surface is too complicated for you, as an approximation just treat the Moon as a flat disk (*cf.* Prob 2 in Chapter 2 of M&P textbook).

Prob 3 (20%)

2. A hypothetical structure, called a "Dyson sphere", is a structure that an advanced civilization might build to harvest as much energy as possible from the star of their "solar system". To explain the concept, note that for our solar system most of the radiative energy emitted by the Sun escapes to the vast universe, with only an extremely small fraction of it being intercepted by the Earth for the use of human beings. As envisioned by F. Dyson*, to maximally harvest solar energy, one might build a spherical shell, for instance with a radius of 1 A.U. and centered at the Sun, that completely encloses the Sun. Such a structure, if it exists, would have a much lower temperature than the Sun itself such that the entire solar system would appear (to an observer outside the solar system) to emit infrared radiation. Dyson suggested a search of "infrared stars" as a way to detect extraterrestrial intelligent life.

Suppose that such a structure with a radius of 1 A.U. is built for our solar system. Moreover, assume that the spherical shell is very thin and highly conductive such that its inner surface (facing the Sun) and outer surface (facing the universe for aliens to observe) have the same temperature. The material used to build the shell has an albedo of 0.2 (i.e., 20 % of the incident sunlight is bounced off the shell, while 80% is absorbed by it). **(a)** What would be the temperature, in °K, of the spherical shell at radiative equilibrium? **(b)** Using Wien's displacement law, what would be the peak wavelength, in μm, of the radiation emitted by this Dyson sphere? **(c)** What would be the surface temperature of the Dyson sphere if its radius is 3 A.U. instead of 1 A.U.?

* The original article is F. J. Dyson, 1960, *Science*, Vol. 131, pp. 1667-1668.