

MAE 578, Spring 2015 Homework #4

1. (35%) In the vicinity of the City of Tempe, marked by a circle in Fig. 1, a rectangular grid of meteorological stations was set up to collect observations of temperature, velocity, etc. The spacing of the stations is 30 km in both x- and y-direction. At 9 PM of a certain day, the observation of the flow at the vertical level of $z = 3$ km indicates that (see illustration in Fig. 1) (i) The horizontal velocity is uniform in space and points to the northeast with a speed of 5 m/s. The wind vectors (red arrows) form a 45° angle with the x-axis, and (ii) The temperature contours (bold gray lines) are straight and equally spaced lines that form an acute angle, θ , with the x-axis where $\tan \theta = 0.5$. The temperature at Tempe is 10°C . Moreover, from the observations of temperature at other vertical levels, it has been determined that (iii) The vertical lapse rate over the area of interest is $\Gamma \equiv -\partial T/\partial z = 8^\circ\text{C}/\text{km}$ at the $z = 3$ km level. (The atmosphere is statically stable.)

From the observations of the horizontal velocity at other vertical levels, your colleague has performed a vertical integration of the horizontal wind divergence to determine that (iv) The vertical velocity at $z = 3$ km is approximately -5 cm/s (a subsidence) uniformly over the area of our interest. Lastly, your colleague has also performed a radiative transfer calculation to estimate that (v) The radiative cooling rate (\dot{Q}/c_p is negative) of the atmosphere at $z = 3$ km is approximately $2^\circ\text{C}/\text{hour}$ uniformly over the area of our interest. Using the information from (i)-(v) and ignore the effect of moisture, try to make a prediction of the temperature at $z = 3$ km for Tempe at 10 PM (i.e., an hour later) using the standard temperature equation,

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + (\Gamma - \Gamma_d)w + \dot{Q}/c_p, \quad \text{Eq. (1)}$$

where $\Gamma \equiv -\partial T/\partial z$ is the vertical lapse rate of the environmental temperature and $\Gamma_d \equiv g/c_p$ is the dry adiabatic lapse rate. To simplify the calculation, you may further assume that (vi) The velocity field remains the same over the one hour period from 9 PM - 10 PM.

Note: In reality, most weather observation and forecast systems use pressure (instead of height) as the vertical coordinate. For the purpose of this exercise this is not critical. We assume that the vertical level shown in Fig. 1 is a constant- z level and the "w" in Eq. (1) is the vertical velocity in z -coordinate. The (u,v) given in the problem is the horizontal velocity at this constant- z level.

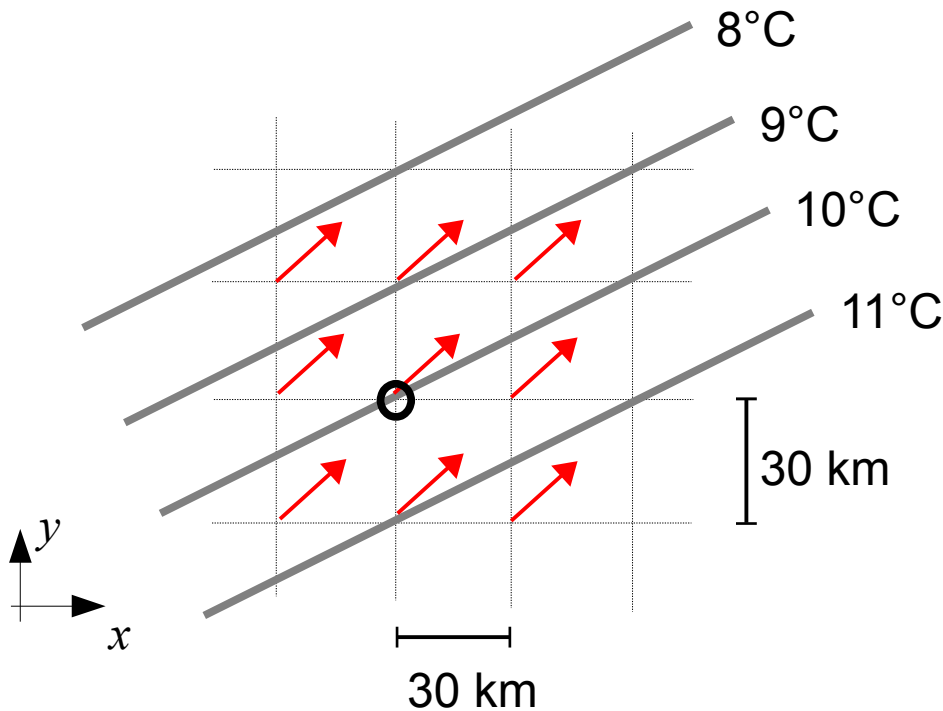


Fig. 1

2. (35%) Solve Prob 6 in Chapter 7 of the textbook. You do not need to answer the question in the last paragraph ("What vertical displacement ...") of that problem.
3. (10%) Solve Prob 3(a) in Chapter 6 of the textbook. You do not need to answer Part (b) of that problem. (You might find the schematic diagram in Fig. 6.18 useful for setting up the solution for the problem. Be aware that that diagram is not drawn to scale.)
4. (10%) Solve Prob 5 in Chapter 6 of the textbook. You do not need to answer the last question ("What analogies can you draw ...") of that problem.
5. (10%) A plaque that introduces the Foucault pendulum in the PSF Building (that we visited recently) states that the vertical plane of swing of the pendulum rotates clockwise with a period of 43 hours and 34 minutes. Try to explain where this number comes from. The plaque, pictured below, also provides other pieces of information such as the latitude of Tempe, length of the cable, mass of the ball, etc. They may or may not be relevant but are included here for your reference.

