MAE578, Spring 2015 Homework #5

1. This problem is concerned with the structure of large-scale horizontal wind field within planetary boundary layer. Recall that the "three-way" balance we discussed in class can be written as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f\,v + K\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f\,u + K\frac{\partial^2 v}{\partial z^2} \quad , \qquad (2)$$

where K is the "eddy viscosity" and all other notations are standard. For this problem (both Part (a) and (b)), we consider the simpler case when there is no pressure gradient force in the x-direction while the pressure gradient force is uniform in the y-direction. (Note that the pressure gradient force in the y-direction could still vary with height, as is the case in Part (b).) We will consider the pressure field as given and use that information to determine the wind field.

(a) (25%) Re-derive and verify the Ekman solution we discussed in class by obtaining the solution for u(z) and v(z) from Eqs. (1) and (2), plus the boundary conditions,

(i) $(u, v) \rightarrow (u_g, 0)$ as $z \rightarrow \infty$ $\quad$ ($u_g \equiv -(\partial p/\partial y)/\rho f$; It is understood that $v_g = 0$.)
(ii) $(u, v) = (0, 0)$ at $z = 0$.

Moreover, for this part only, assume that the pressure gradient force in y-dirction is independent of height. Write your solution in terms of u_g, K, and f. Make a plot of the vertical profiles of u(z) and v(z). For this plot, it is recommended that (u,v) be rescaled with u_g and z be rescaled with $(2K/f)^{1/2}$. In addition, make a plot of the "Ekman spiral" in the u-v plane by tracking the tip of the horizontal wind vector with increasing height. Please also superimpose wind vectors at selected heights.

(b) (25%) Instead of assuming that the y-component of pressure gradient force is independent of height, we now consider the case when it increases with height. Specifically, it is given as

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} \equiv G\left(1 - e^{-z/H}\right) \quad ,$$

where G and H are two adjustable parameters. The corresponding boundary conditions are

(i) $(u, v) \rightarrow (G/f, 0)$ as $z \rightarrow \infty$
(ii) $(u, v) = (0, 0)$ at $z = 0$.

Solve this problem and express the solution (u(z), v(z)) in terms of G, H, K, and f. At 30°N and under the parameter setting with $G/f = 5$ m/s and $(2K/f)^{1/2} = 1000$ m, plot the vertical profiles of u(z) and v(z) for the three cases with $H = 500$, 1000, and 2000 m. (Note that G/f is u_g as $z \rightarrow \infty$, and $(2K/f)^{1/2}$ is the typical scale of the thickness of planetary boundary layer from the classic Ekman solution in Part (a).) Discuss how the value of H affects the solution. For example, discuss how the thickness of boundary layer scales with $(2K/f)^{1/2}$ and/or H for the cases with $H \gg (2K/f)^{1/2}$, $H \approx (2K/f)^{1/2}$, and $H \ll (2K/f)^{1/2}$.

2. (30%) We have previously learned about the "moist static energy", $E_S \equiv C_P T + gz + Lq$ (see Sec. 4.5.2), which can be regarded as the energy (per unit mass) of an air parcel in the absence of any motion. Taking into account atmospheric motion, the total energy would be $E = E_S + E_K$, where $E_K \equiv (1/2)\,|\mathbf{v}|^2$ is kinetic energy (**v** is the 3-D velocity vector; cf. Eq. (8-14)). Using the collection of figures in Chapter 5 for the climatological mean state of the atmosphere as a function of latitude and height (or pressure), try to estimate the magnitude of the individual components of E at selected latitudes and pressure levels and fill the blanks in the following table. Comment on your results. The purpose of this exercise is for you to become familiar with the climatological state presented in Chapter 5. We will revisit energy balance in Chapter 8.
(Note: For this exercise, please use q instead of q_S in the calculation for E_S.)

	$C_P T$	gz	Lq	$(1/2)\|\mathbf{v}\|^2$	E
@ Equator and 1000 mb level					
@ Equator and 200 mb level					
@ 45°N and 1000 mb level					
@ 45°N and 200 mb level					

All quantities are in m^2/s^2

Note: (1) The height field shown in Fig. 5.13 is the "anomaly", i.e., departure from a certain global-mean value. More precisely, if we denote the anomaly as $z^*(\phi,p)$ (ϕ is latitude) and the global mean as $Z(p)$, then the total height is $z(\phi,p) = z^*(\phi,p)+Z(p)$. It is this total height that should be used for evaluating the "gz" term. For this exercise, let's assume that the global mean $Z(p)$ is 12 km at p = 200 mb and 0 km at p = 1000 mb. (2) Strictly speaking, the **v** for the evaluation of kinetic energy should be the three-dimensional velocity. Since for global-scale circulation the "zonal component" (u) tends to be greater than v and w, for this exercise we will approximate $|\mathbf{v}|^2$ by $|u|^2$, where the magnitude of u can be inferred from its "zonal mean" in Fig. 5.20.

3. (20%) Solve Prob 9 in Chapter 7 of the textbook. The "Winter Pole" in that problem will be regarded as the South Pole. (Note that the Coriolis parameter, *f*, is negative in the Southern Hemisphere.)