Atmospheric "greenhouse effect"
- How the presence of an atmosphere makes Earth's surface warmer

Some relevant parameters and facts (see previous slide sets)

$(S_o/4) \approx 342$ W m^{-2} is the average incoming solar radiative energy per unit area for planet Earth.
($S_o \approx 1368$ W m^{-2} is the Solar constant)

α_p is planetary albedo. In the absence of an atmosphere, it would be the surface albedo.

Stefan-Boltzmann law: A body with temperature T emits radiation with the energy per unit area = σT^4
$\sigma \approx 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is Stefan-Boltzmann constant; T in °K

Earth's atmosphere is nearly transparent to solar (shortwave) radiation
The "greenhouse gases" in the atmosphere do absorb terrestrial (longwave) radiation

Important greenhouse gases: H_2O , CO_2 , CH_4 , etc.

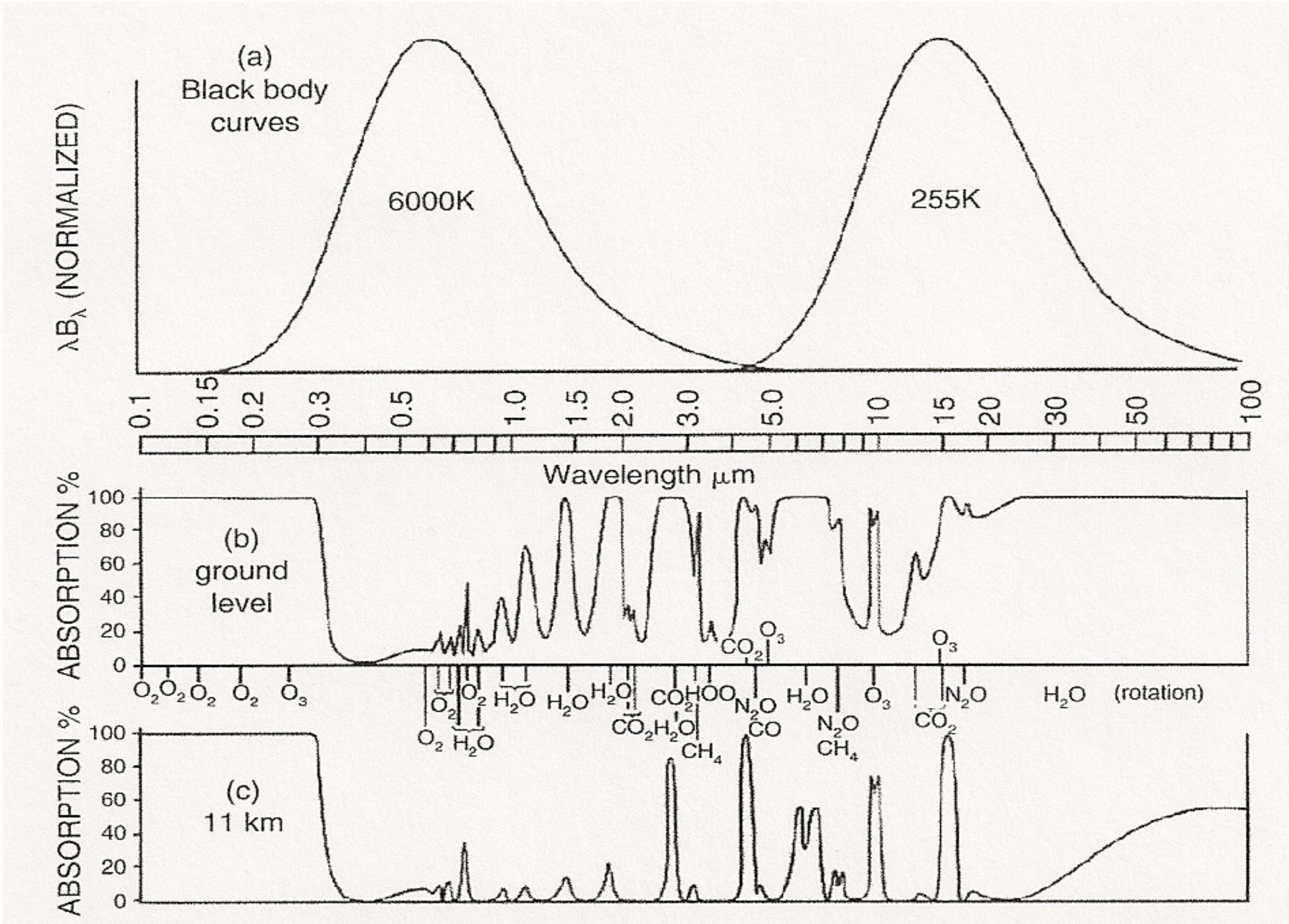


Fig. 2.6 in M&P (yet again)

Case 1. Earth in radiative equilibrium without an atmosphere

The incoming solar energy per unit area is $(S_o/4)\,(1 - \alpha_p)$

At temperature T_E, the energy per unit area radiated by the Earth is (by Stefan-Boltzmann law) $\sigma\, T_E^4$

Let "in = out", $\sigma\, T_E^4 = (S_o/4)\,(1 - \alpha_p)$, we have

$$T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$$

For example, if $\alpha_p = 0.1$, we have $T_E \approx 271.4$ K

Notations (for convenience of discussion):

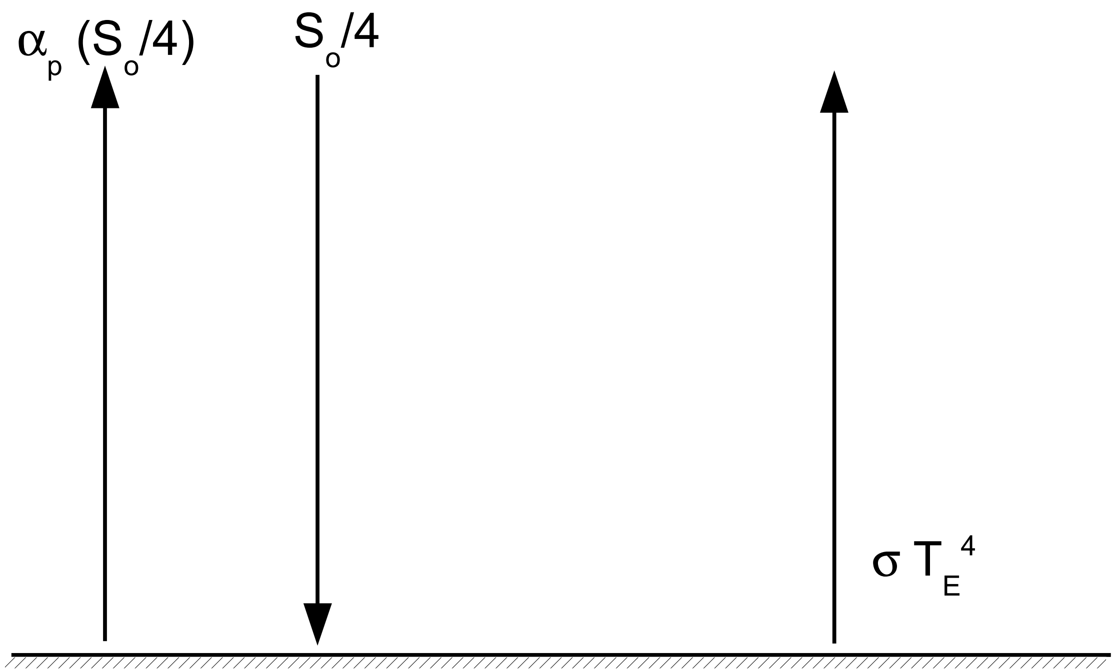
SW : "shortwave radiation" : the radiation directly emitted by the Sun, or the sunlight reflected by the Earth system), $\lambda \sim$ visible band

LW: "longwave radiation" : the radiation emitted by the Earth and/or its atmosphere, $\lambda \sim$ infrared band

SW $\uparrow$: Upward shortwave radiation

LW $\downarrow$: Downward longwave radiation, etc.

In previous example (Earth with no atmosphere):

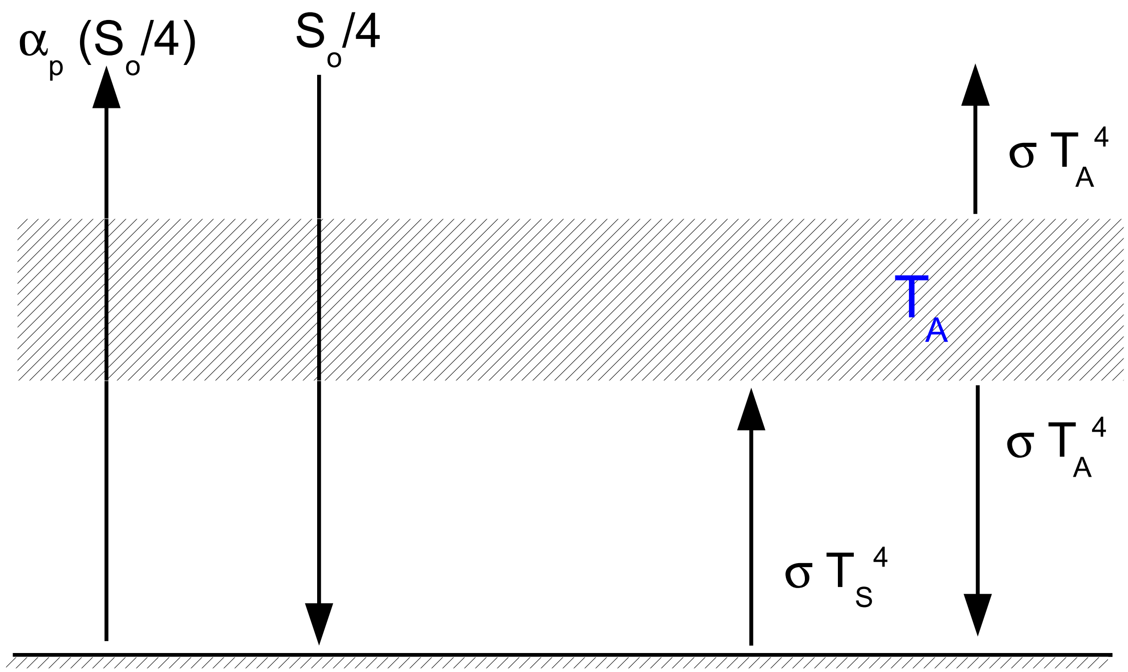


At the surface, SW ↓ = $(1-\alpha_p)$ $(S_o/4)$, LW ↑ = σT_E^4

Radiative equilibrium @ surface: **SW ↓ = LW ↑**

=> $T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$

Case 2. One-layer atmosphere that is transparent to SW but completely absorbing of LW radiation



@ surface: $SW\downarrow = (1-\alpha_p)\,(S_o/4)$, $LW\downarrow = \sigma\, T_A^4$, $LW\uparrow = \sigma\, T_S^4$

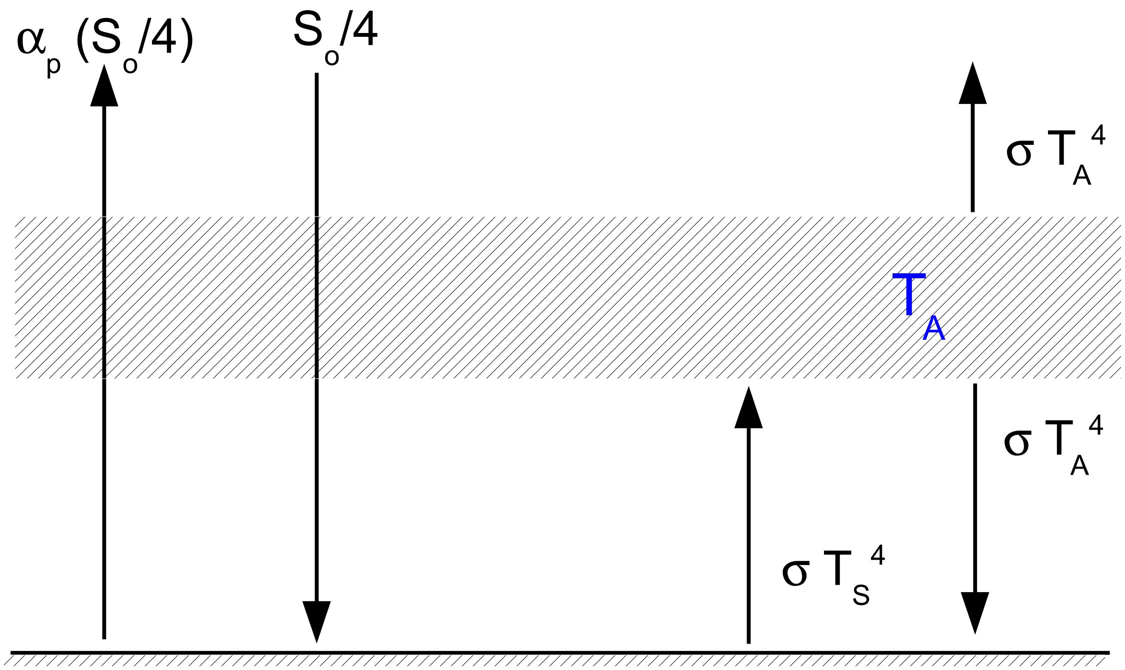
Radiative equilibrium for Earth surface: $\mathbf{SW\downarrow + LW\downarrow = LW\uparrow}$

=> $\sigma\, T_S^4 = (S_o/4)(1 - \alpha_p) + \sigma\, T_A^4$ Eq. (1)

@ top of atmosphere: $SW\downarrow = (1-\alpha_p)\,(S_o/4)$, $LW\uparrow = \sigma\, T_A^4$

Radiative equilibrium for the earth system: $\mathbf{SW\downarrow = LW\uparrow}$

=> $\sigma\, T_A^4 = (S_o/4)(1 - \alpha_p)$ Eq. (2)



$\sigma T_S^4 = (S_o/4)(1 - \alpha_p) + \sigma T_A^4$ Eq. (1)

$\sigma T_A^4 = (S_o/4)(1 - \alpha_p)$ Eq. (2)

$\Rightarrow T_A = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$

and

$2 \sigma T_A^4 = \sigma T_S^4 \Rightarrow T_S = 2^{1/4} T_A$

For example, if $\alpha_p = 0.1$, we have $T_A \approx 271.4$ K and $T_S \approx 322.7$ K (surface is warmer)

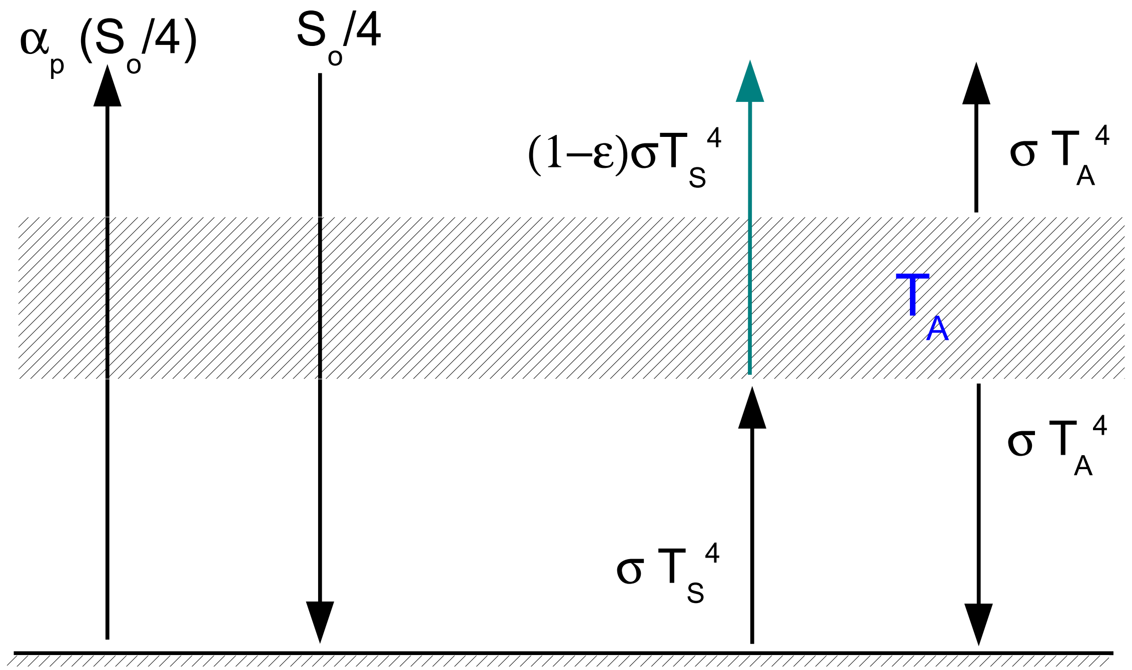
Note :

(1) The T_A is now the effective temperature, T_E, for the Earth system (concerning its emission of radiation back to space). But note that it remains the same as the T_E in the absence of an atmosphere

=> The presence of greenhouse gases in the atmosphere makes the surface of the Earth warmer, but it does not modify the total (Earth surface + atmosphere) longwave radiation that the Earth emit back to space, i.e. LW↑ at the top of the atmosphere. In radiative equilibrium, this quantity depends only on the Solar constant and planetary albedo.

(2) $T_S > T_A$ is always true, regardless of the detail of the greenhouse-gas absorptivity, etc.

Case 3. One-layer atmosphere that is transparent to SW and partially absorbing of LW radiation, with ε being the fraction of IR absorption by the atmosphere



@ surface: $SW\downarrow = (1-\alpha_p)(S_o/4)$, $LW\downarrow = \sigma T_A^4$, $LW\uparrow = \sigma T_S^4$

$$\Rightarrow \sigma T_S^4 = (S_o/4)(1-\alpha_p) + \sigma T_A^4 \qquad \text{Eq. (1)}$$

@ top of atmosphere: $SW\downarrow = (1-\alpha_p)(S_o/4)$, $LW\uparrow = \sigma T_A^4 + (1-\varepsilon)\sigma T_S^4$

$$\Rightarrow \sigma T_A^4 + (1-\varepsilon)\sigma T_S^4 = (S_o/4)(1-\alpha_p) \qquad \text{Eq. (2)}$$

(1) & (2) $\Rightarrow T_S = (2/\varepsilon)^{1/4}\, T_A > T_A$ (Note that $0 \le \varepsilon \le 1$)

Interpreting $T_S = (2/\varepsilon)^{1/4}\, T_A$ or $T_A = (\varepsilon/2)^{1/4}\, T_S$:

Recall that ε is the infrared absorptivity of the atmosphere, $0 \le \varepsilon \le 1$

For example, if ε = 0.8, 80% of the infrared radiation emitted by Earth's surface is absorbed by the atmosphere, while 20% of it passes through the atmosphere to escape to space

$\varepsilon = 0.8 \Rightarrow T_A = (0.8/2)^{1/4}\, T_S = 0.795\, T_S$

$\varepsilon = 0.2 \Rightarrow T_A = (0.2/2)^{1/4}\, T_S = 0.562\, T_S$

Smaller ε => Colder atmosphere

This is solely expected, since the atmosphere needs to have an intake of energy (by absorbing LW radiation) to keep itself warm.

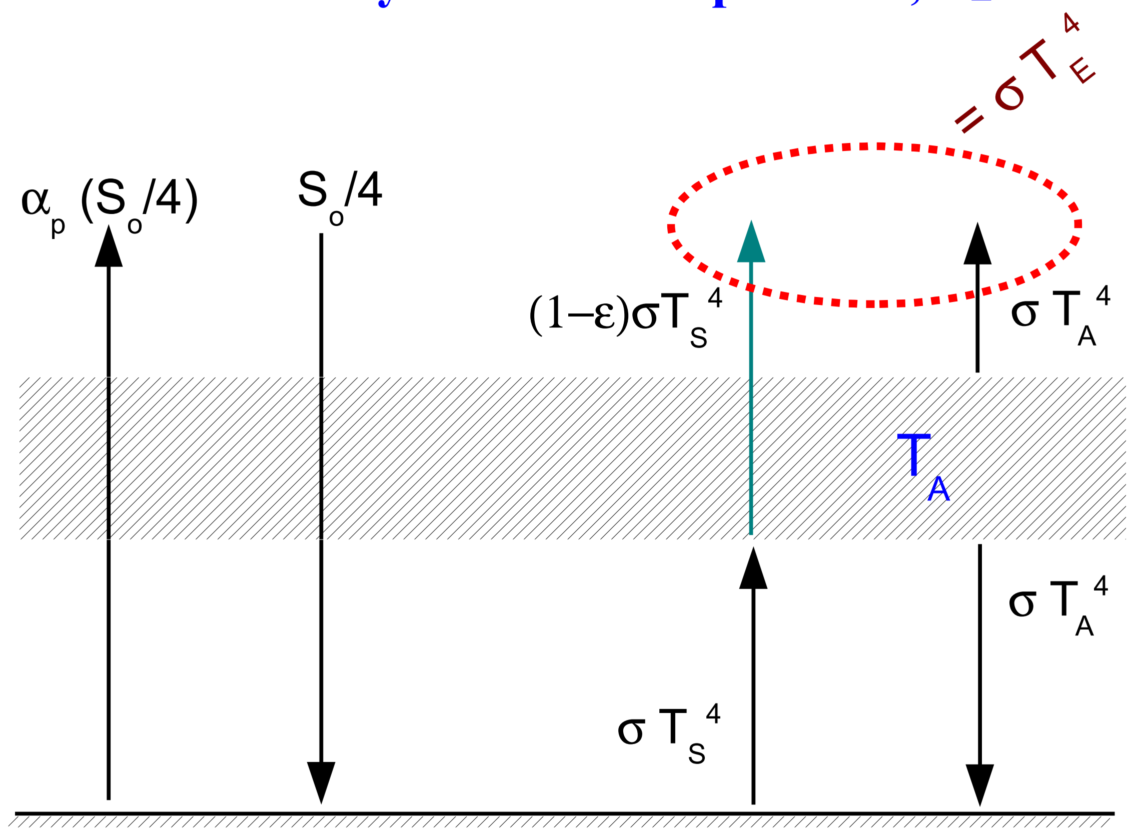
(We have already assumed that the atmosphere is transparent to SW radiation, therefore LW radiation is the only energy source for the atmosphere)

As $\varepsilon \longrightarrow 0$, $T_A \longrightarrow 0$

When the atmosphere is transparent to both SW and LW radiation, it has zero gain of radiative energy. Yet, recall that any object that has a non-zero temperature would emit radiation by Stefan-Boltzmann law. Therefore, the atmosphere will keep emitting LW radiation and getting colder (until it reaches 0°K, then an equilibrium is reached with zero in, zero out)

In reality, an atmosphere of such nature (one filled only with gases that do not absorb LW and SW radiation, e.g., nitrogen) will not go to 0°K. This atmosphere can still maintain a finite temperature by heat conduction (the atmosphere is in contact with the surface of the planet) and convective heat transfer through atmospheric motion. Recall that radiative transfer is but one of the three ways to move heat around.

Planetary emission temperature, T_E



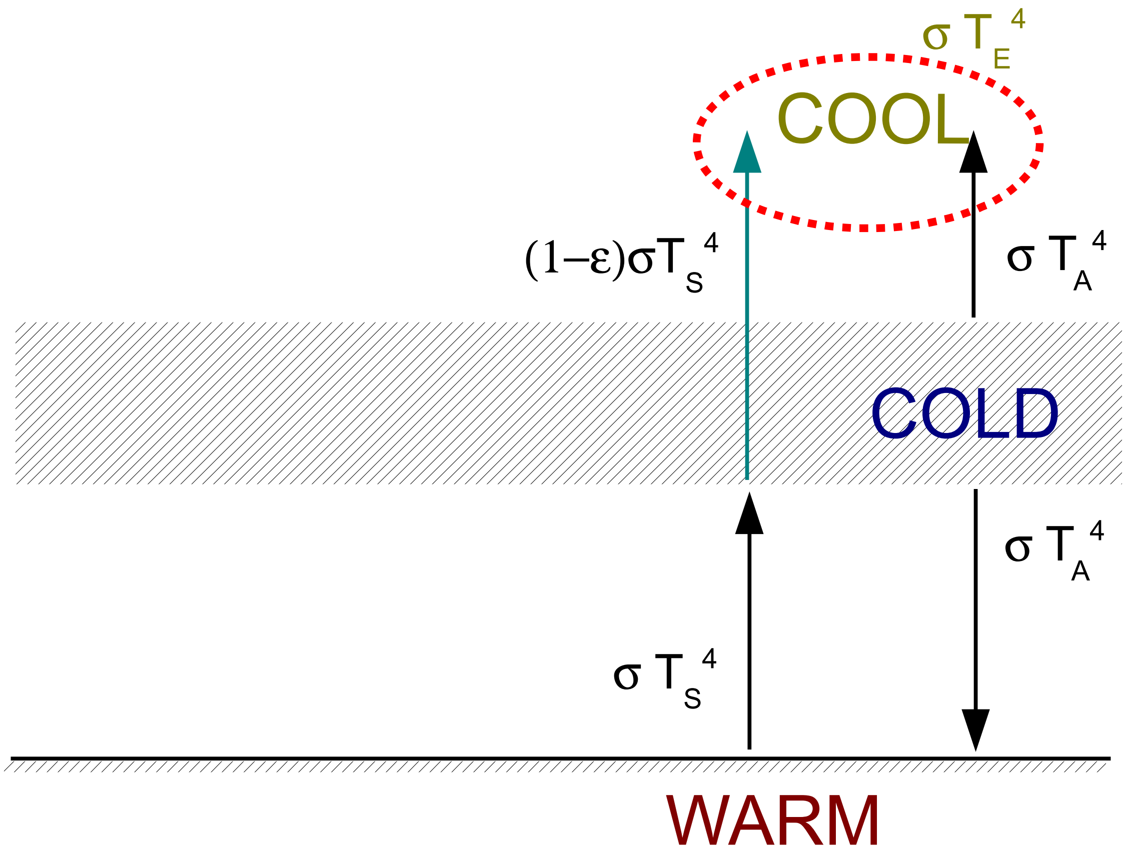
Collect all outgoing LW radiation at the top of the atmosphere and convert it to an "equivalent temperature" by Stefan-Boltzmann relationship, this temperature is our "planetary emission temperature", T_E.

For Case 3, $\sigma T_E^4 = \sigma T_A^4 + (1-\varepsilon)\sigma T_S^4$

Using $T_S = (2/\varepsilon)^{1/4} T_A$, we find $T_A = [\varepsilon/(2-\varepsilon)]^{1/4} T_E \,\frac{c}{\lambda}\, T_E$ and $T_S = [2/(2-\varepsilon)]^{1/4} T_E \,{}^{24}_{12}\, T_E$

=> $T_A \leq T_E \leq T_S$ (if $\varepsilon < 1$)

We have $T_A \le T_E \le T_S$, because the "planetary emission temperature" T_E represents the weighted average of the "warmer" radiation from the surface and the "colder" radiation from the atmosphere



$$\sigma T_E^4 = (S_o/4)(1 - \alpha_p)$$

$$\Rightarrow T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$$

The planetary emission temperature (or the total outgoing LW radiation at the top of the atmosphere) depends only on the Solar constant and planetary albedo. **The effect of atmospheric greenhouse gases modifies surface temperature and atmospheric temperature profile but it does not alter T_E or LW↑ @TOA**

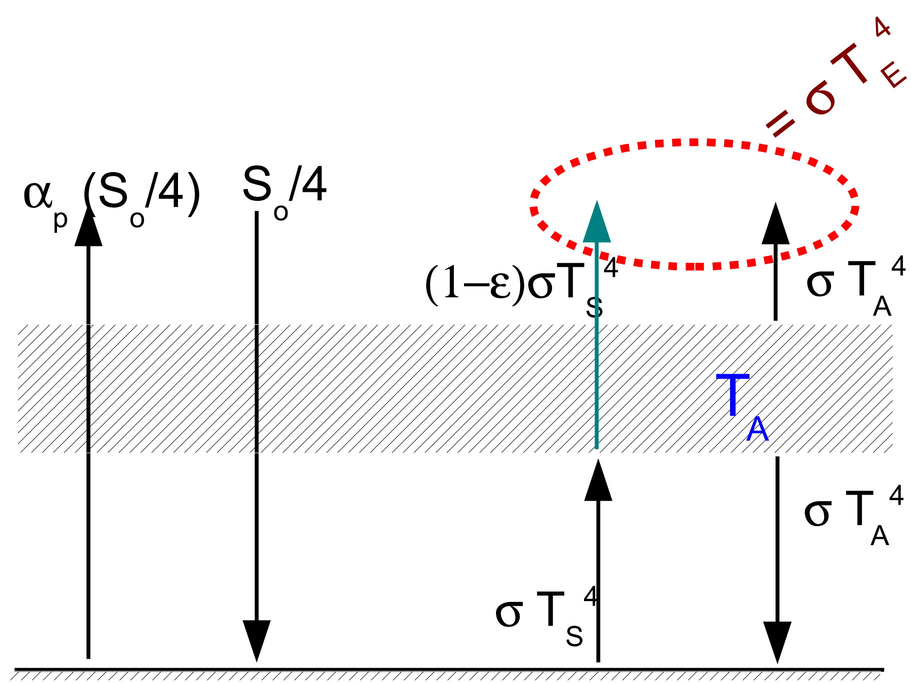
For example, if we double the amount of CO_2 in the atmosphere, Earth's surface will become warmer but the T_E or LW↑ @TOA will remain the same as what it was under 1 x CO_2. (This implies cooling in the stratosphere, to compensate for the warming in the lower atmosphere and surface.)

If the doubling of CO_2 leads to a change in (say) cloud cover, then planetary albedo could change, potentially leading to a change in T_E or LW↑ @TOA. This is another story ...

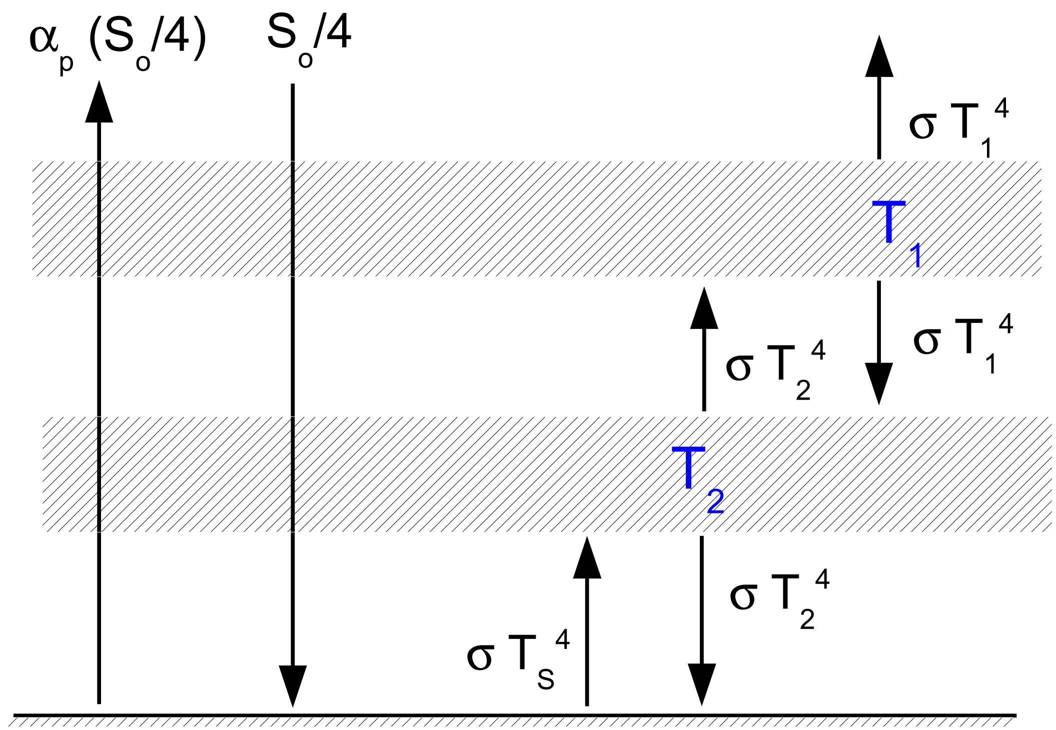
Summary for Case 3

$T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$
$T_A = [\varepsilon/(2-\varepsilon)]^{1/4}\ T_E$
$T_S = [2/(2-\varepsilon)]^{1/4}\ T_E$

For example, for $S_0/4 = 342\ \mathrm{W\ m^{-2}}$, $\alpha_p = 0.1$, $\varepsilon = 0.8$, we have $T_E = 271.4$ K, $T_A = 245.2$ K, and $T_S = 308.3$ K



Case 4. Two layers of atmosphere, both with 100% IR absorptivity (but remain transparent to SW radiation) $T_E = T_1$ in this case



@ TOA, $\sigma T_1^4 = (S_o/4)(1 - \alpha_p)$ $T_1 = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$ Eq. (1)

@ Surface, $\sigma T_S^4 = \sigma T_2^4 + (S_o/4)(1 - \alpha_p) = \sigma T_1^4 + \sigma T_2^4$ Eq. (2)

For layer 2, $\sigma T_1^4 + \sigma T_S^4 = 2\,\sigma T_2^4$ Eq. (3)

Combine (1)-(3): $T_S = 3^{1/4}\,T_1$, $T_2 = 2^{1/4}\,T_1$ => $\mathbf{T_S > T_2 > T_1}$

Temperature decreases with height

Case 5
If we modify Case 4 by adding more layers, the conclusion will remain the same that temperature decreases with height, while the top layer always has the temperature of $T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$.

In this case, the surface becomes warmer if we add more (totally absorbing) layers. In other words, as the atmosphere becomes thicker (having more greenhouse gases in it - only greenhouse gases count), the surface of the planet becomes warmer (cf. Prob 6 of Chap 2 in M&P textbook).

Try to verify that these points are true.

Case 6

If we modify Case 4 or 5 by assuming partial IR absorption ($\varepsilon < 1$) for the atmospheric layers, the resulted temperature profile will still decrease with height (surface being the warmest). In this case, like Case 3, the upper layers will be colder than T_E while the lower layers and surface will be warmer than T_E, where $T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$ is the planetary emission temperature.

Try to verify that these points are true.

Summary (radiative equilibrium temperature profile)