# Adiabatic process and potential temperature

Adiabatic process in liquid fluid (~ incompressible) vs. the atmosphere (compressible)

Liquid: Consider a 2-layer fluid; A parcel of fluid is carved out of the lower layer and lifted adiabatically to the upper layer. In this process, the environmental pressure decreases and so does the pressure inside the parcel (as it adjusts to the environmental pressure). But, because the density of a liquid fluid depends very weakly on pressure, the density (or volume) of the parcel stays almost the same under the reduced pressure. **Density of the parcel is nearly conserved** in the process. One can immediately conclude that the lifted parcel is heavier than its environment if the lower layer (where the parcel came from) is denser than the upper layer.
=> Static stability can be determined by the **environmental density profile**
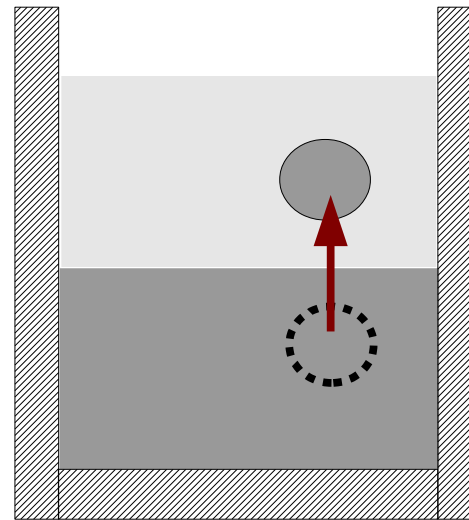
The atmosphere

Stratified in the vertical; $T(z)$, $p(z)$, $\rho(z)$
Pressure decreases with height (hydrostatic)
Ideal gas: $\rho = p/RT$, density can vary substantially with pressure and temperature

As an air parcel is adiabatically lifted to a new height, its density changes. All (ρ, p, T) of the parcel change $\Rightarrow$ more complicated to determine whether the lifted parcel is heavier or lighter than the environment
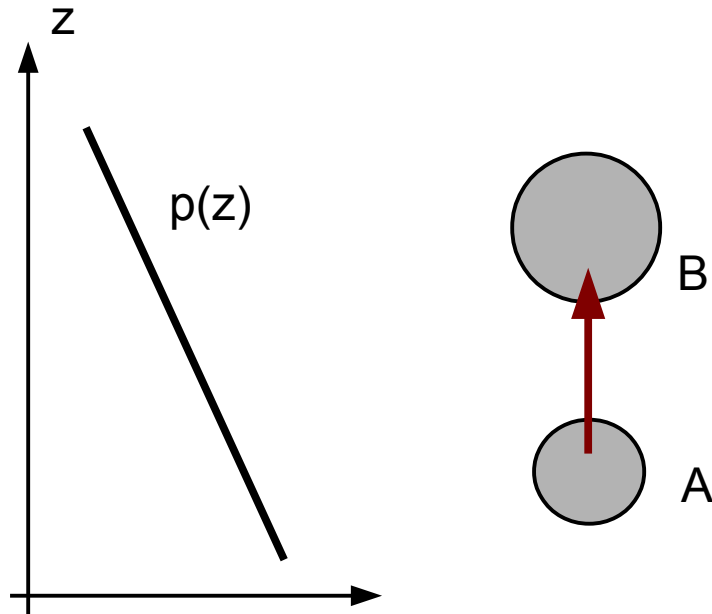
Is there a conservative quantity following an air parcel that is lifted adiabatically?

Clearly, the entropy of the parcel is conserved. Recall that $dS = dQ/T$. Since $dQ = 0$ (adiabatic; no heat exchange with the environment), $dS = 0 \implies S_{initial} = S_{final}$

For most applications, we prefer to use a related quantity called **potential temperature**, which is also conserved following an adiabatic process

Consider a thermally insulated chamber capped by a frictionless piston that is initially resting at the surface; the pressure inside the chamber equals the environmental surface pressure, p_S. The apparatus is then attached to a balloon that lifts it to a higher level with a lower environmental pressure, $p < p_S$. Along the way, the pressure inside the chamber adjusts to the environmental pressure (this is important to remember). We have previously found that the temperature inside the chamber is

$$T = T_S (p/p_S)^{R/Cp} < T_S \quad \text{(air parcel cools)}$$

If we now punch a hole in the balloon to let the apparatus fall back (adiabatically) to the level of $p = p_S$, the temperature in the chamber will be restored to T_S.
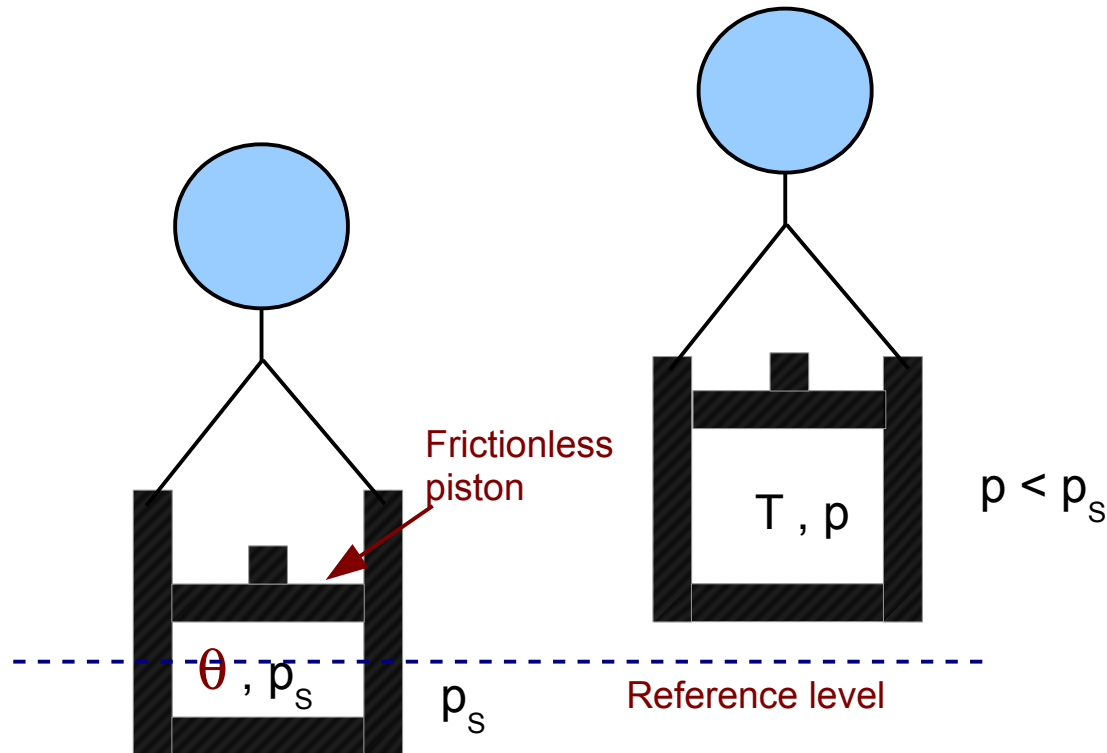

Frictionless piston

T , p

$p < p_S$

T_S , p_S

p_S

Reference level
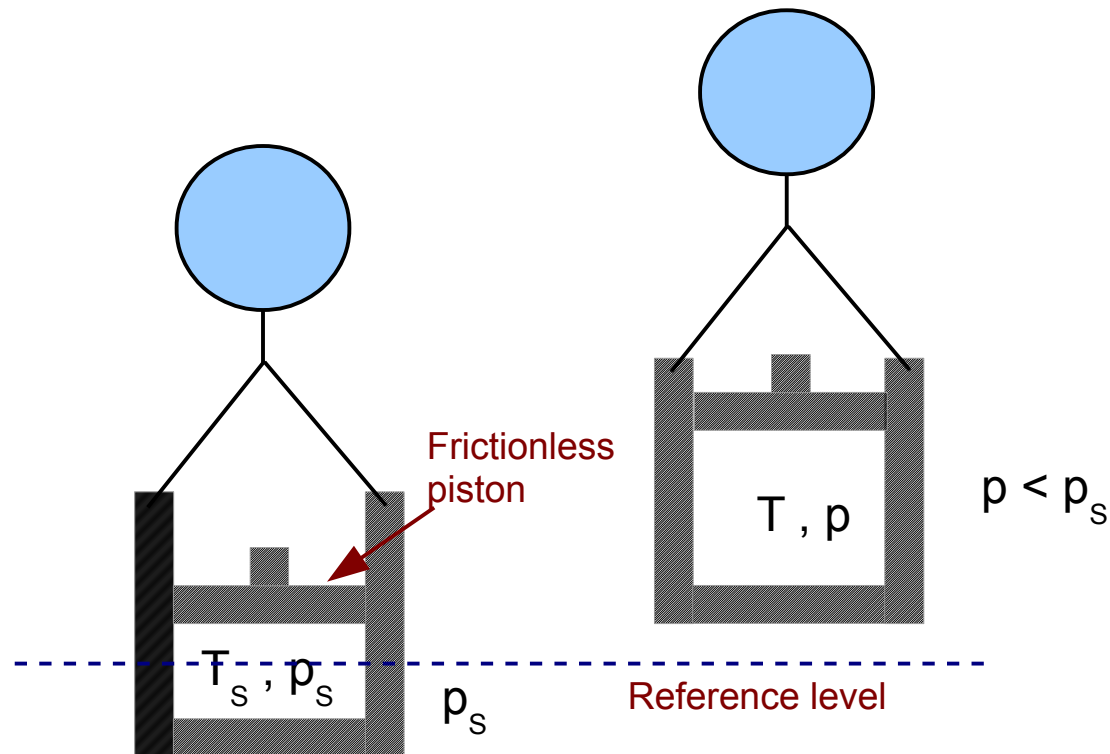
(continued)

In that sense, T_S is a conservative quantity for the air parcel (inside the chamber) that moves up and down adiabatically. We will define it as the "potential temperature", θ.

$$\theta \equiv T_S = T \, (p_S/p)^{R/C_p} \quad .$$

Beware that we have switched the positions of T and T_S from the formula in previous page.

Frictionless piston

θ , p_S

p_S

T , p

$p < p_S$

Reference level

We usually choose a constant sea-level pressure (e.g., p_S = 1000 mb) as the reference pressure to define potential temperature, but any choice of p_S will do.

Given T(z) and p(z) for the environment, we can also define the potential temperature profile for the environment,

$$\theta(z) \equiv T(z) \, (p_S/p(z))^{R/C_p}$$

This can be understood in the following sense: If we carve an air parcel out of the environment at level z and push it down (or lift it up) adiabatically to the reference level, p_S, $\theta(z)$ is the temperature of the parcel when it reaches the reference level.

Although we invoke the parcel method to understand the definition of environmental potential temperature, the $\theta(z)$ here is a *field* that does not have to be adhered to a specific parcel.

Potential temperature and entropy

Recall that $dS \equiv dQ/T$, and (from 1st law of thermodynamics) $dQ = C_P\, dT - \alpha\, dp$

$\Rightarrow$ For an ideal gas (invoking $p\alpha = RT$), $dS = C_P\, d\ln T - R\, d\ln p$

By the definition of potential temperature, we have

$$\ln\theta = \ln T + (R/C_P)\,(\ln p_S - \ln p)$$
$$= \ln T - (R/C_P)\ln p + \text{constant}$$

$$\Rightarrow\ d\ln\theta = d\ln T - (R/C_P)\ln p.$$

Therefore,

$$\color{red}{dS = C_P\, d\ln\theta}$$

Following an adiabatic process, $dQ = 0 \rightarrow dS = 0 \rightarrow d\theta = 0$

We like θ because it can be directly related to temperature. Later, we will show that the "perturbation" potential temperature can be related to "perturbation" density. (This would help us further understand the criterion of static stability in terms of potential temperature.)