

Hydrostatic, non-hydrostatic, and anelastic model*Background*

In our discussion on the governing equations used in numerical weather and climate prediction, it was noted that most (if not all) of the numerical models used in real applications today are either hydrostatic or fully non-hydrostatic. The latter admits acoustic waves which significantly reduces the required time step size, Δt , for the numerical integration. This is one of the reasons that, until recently, the "climate" models used for long-term integration are almost exclusively hydrostatic. At the same time, many regional models for short-term weather prediction are now fully non-hydrostatic.

The hydrostatic and fully non-hydrostatic versions of the "primitive equations" are at the two ends of the spectrum in terms of the level of complexity of the model. In the history of the development of atmospheric models, attempts have been made to find alternatives that are more accurate than the hydrostatic model (e.g., the vertical component of the momentum equation is prognostic instead of diagnostic) but still exclude sound waves. One of such systems is the *anelastic* equation set (Ogura and Phillips 1962). Your task in this assignment is to survey and explore the properties of this class of equations.

Main tasks

Read the original and review papers listed below on the anelastic and related approximations and perform a literature survey to find more articles on the subject. From the literature survey, address the following points:

- (1) Describe the structure of the anelastic and related systems and discuss how they differ from the hydrostatic and fully non-hydrostatic systems. (Describe how the momentum, thermodynamic, and continuity equations in those systems differ from each other.)
- (2) On what spatial and temporal scales is the anelastic approximation valid? What phenomena may be ideal to use an anelastic model to simulate?
- (3) In the derivation of the anelastic and related systems, what approximation(s) were made to filter out sound waves? Note that the generation of sound waves is related to the compressibility of the air, quantified by the change in perturbation density in response to the change in perturbation pressure, $(\partial\rho/\partial p)$.
- (4) Survey the literature to find examples (if any) of published results of numerical simulations that used the anelastic equation set. Why is the anelastic system not widely used in practice (compared to the hydrostatic and fully non-hydrostatic systems) for weather and climate simulations? Is there any inherent difficulty for the numerical implementation of the anelastic system for practical applications?

References

Do not restrict yourself to the following 3 papers. They are merely the starting points of the literature survey.

Ogura, Y., and N. A. Phillips, 1962: Scale analysis of deep and shallow convection in the atmosphere, *J. Atmos. Sci.*, **19**, 173-179

Lilly, D. K., 1996: A comparison of incompressible, anelastic, and Boussinesq dynamics, *Atmos. Res.*, **40**, 143-151

Durrán, D. R., and A. Arakawa, 2007: Generalizing the Boussinesq approximation to stratified compressible flow, *C. R. Mecanique*, **335**, 655-664

In addition to the three main references, the following documentations (one for a hydrostatic global model, the other for a non-hydrostatic regional model) provide some detail of the numerical architecture of hydrostatic and non-hydrostatic models:

Kiehl, J.T., J.J. Hack, G.B. Bonan, et al., 1996: Description of the NCAR Community Climate Model (CCM3). NCAR Technical Note NCAR/TN-420+STR, DOI: 10.5065/D6FF3Q99.

Skamarock, W., J.B. Klemp, J. Dudhia, et al., 2008: A Description of the Advanced Research WRF Version 3. NCAR Technical Note NCAR/TN-475+STR, DOI: 10.5065/D68S4MVH.

All of the above papers and documentations are publicly available. (The papers are available through ASU library online.) Please contact the instructor if you have difficulty locating the papers.

General instruction for the final report

The suggested length of the term paper is 15 pages, single space, excluding figures, tables, and references. The total length (including all elements) is expected to not exceed 30 pages. This is however not a strict requirement. A shorter or longer paper is acceptable as long as it is of good quality. All figures must be professionally made (e.g., by Matlab). Hand-drawn figures are not acceptable except for simple schematic diagrams for illustrating a concept. No electronic submission is accepted. Instructor will collect hard copy of the final reports at a designated time, to be announced later. Late submission will not be accepted.