MAE 578, Spring 2017 Homework #1

One homework set ≈ 10% of the total score for the semester. We expect to have six homework assignments. Hard copy of report is due at the start of class on the due date. Collaboration is not allowed for this assignment.

Prob 1. (40%)
Consider a three-layer model of the atmosphere as illustrated in Fig. 1. All atmospheric layers are transparent to solar radiation. All three layers have an infrared absorptivity (and emissivity) of ε. The surface albedo for solar radiation is α. The effective solar input from the top of the atmosphere is $S_0/4$, where $S_0 = 1368$ W m^{-2} is the solar constant. Based on radiative energy balance, calculate and plot the temperature profile, (T_s, T_1, T_2, T_3), for the three cases with ε = 0.2, 0.4, and 0.6. Please collect all three curves in a single plot.

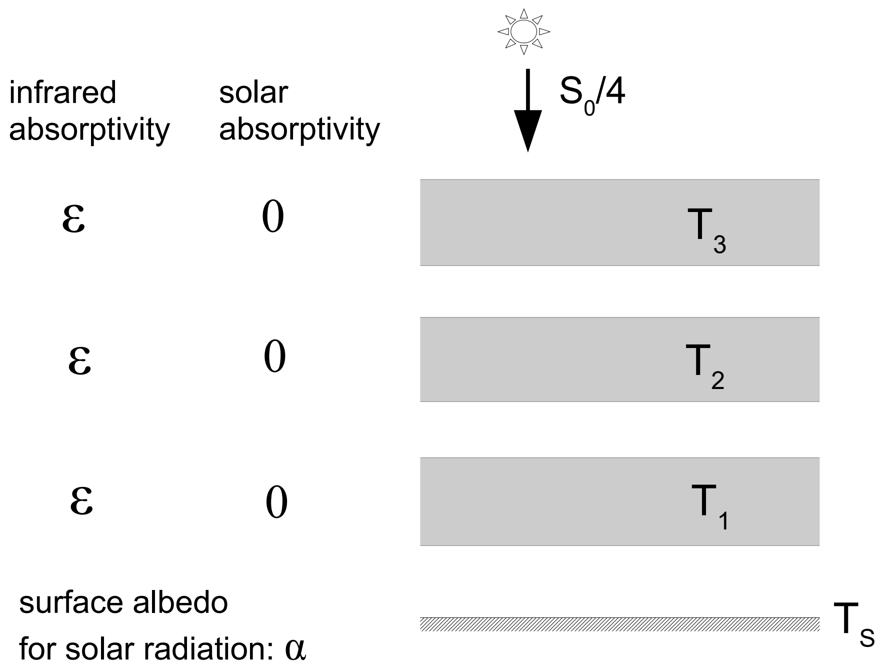

Fig. 1

Prob 2. (20%)
A hypothetical structure, called a "Dyson sphere", is a structure that an advanced civilization might build to harvest as much energy as possible from the star of their "solar system". To explain the concept, note that for our solar system most of the radiative energy emitted by the Sun escapes to the vast universe, with only an extremely small fraction of it being intercepted by the Earth for the use of human beings. As envisioned by F. Dyson*, to maximally harvest solar energy, one might build a spherical shell, for instance with a radius of 1 A.U. and centered at the Sun, that completely encloses the Sun. Such a structure, if it exists, would have a much lower temperature than the Sun itself such that the entire solar system would appear (to an observer outside the solar system) to emit infrared radiation. Dyson suggested a search of "infrared stars" as a way to detect extraterrestrial intelligent life.

Suppose that such a structure with a radius of 1 A.U. is built for our solar system. Moreover, the spherical shell is very thin and highly conductive such that its inner surface (facing the Sun) and outer surface (facing the universe for aliens to observe) have the same uniform temperature. The material used to build the shell has a reflectivity of 0.1 for solar radiation. It otherwise has an emissivity of 1.0 for IR radiation. **(a)** What would be the temperature, in °K, of the spherical shell at radiative equilibrium? **(b)** Using Wien's displacement law, what would be the peak wavelength, in μm, of the radiation emitted by this Dyson sphere? **(c)** What would be the surface temperature of the Dyson sphere if its radius is 2 A.U. instead of 1 A.U.?

* The original article is F. J. Dyson, 1960, *Science*, Vol. 131, pp. 1667-1668.

Prob 3 (40%)
An idealized temperature profile for the atmosphere is given as

$$T(z) = T(0) - \alpha z \,, \qquad 0 < z \le 10 \text{ km}$$
$$T(z) = T(10 \text{ km}) + \beta (z - 10 \text{ km}) \,, \qquad 10 \text{ km} < z \le 30 \text{ km} .$$

where both α and β are positive. This represents the decrease of temperature with height in the troposphere and increase of temperature with height in the stratosphere. See Fig. 2.
(a) Assume that the atmosphere is in hydrostatic balance. Let α = 6 °K/km, β = 3 °K/km, surface temperature T(0) = 300°K, and surface pressure p(0) = 1000 hPa (1 hPa = 100 Pascal), calculate and plot the profiles of T(z), p(z), and ρ(z) (p and ρ are pressure and density) over the range of $0 \le z \le 30$ km. For the calculation, you may set g = 9.8 m s^{-2} and R = 287 J kg^{-1} $°K^{-1}$ (g is gravitational acceleration and R is ideal gas constant).
(b) Repeat (a) but consider α = 11 °K/km. Compare the vertical profiles of T, p, and ρ in this case with their counterparts in (a).
(c) For the case in (a), what is the thickness (in meters) of the atmosphere between the 900 hPa and 500 hPa levels? What is the pressure at z = 30 km?
(d) In general (for any given α and β), if we keep the formula for T(z) intact and fix p(0) but decrease T(0) by 1°K (effectively shifting the temperature profile by a constant 1 °K - see gray lines in Fig. 2), will the pressure at z = 30 km increase or decrease? Interpret your result physically.

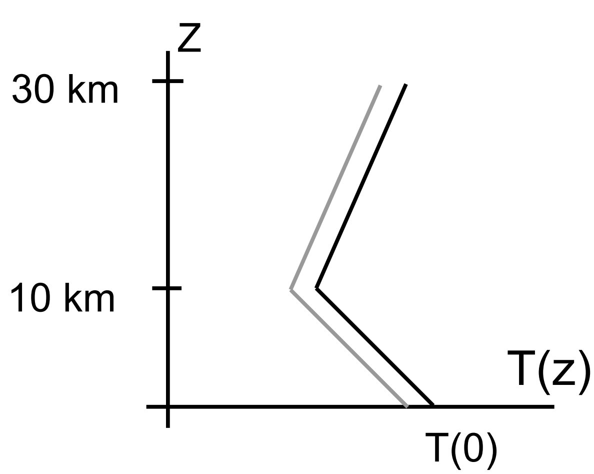

Fig. 2