## MAE578, Spring 2017 Homework \#2

This assignment is equivalent to "one and a half" of a regular homework and will account for $15 \%$ of the total score for the semester. Collaboration is not allowed.

Prob 1. (15\%) An air parcel has an initial temperature of $28^{\circ} \mathrm{C}$ and initial specific humidity of $13 \mathrm{~g} / \mathrm{kg}$. It is initially located at the 1000 hPa pressure level. (a) What is the relative humidity, $\gamma$, and the partial pressure of water vapor, $e$, for this air parcel? (b) If we adiabatically lift this air parcel to the 920 hPa level, will adiabatic cooling be enough to trigger condensation in the parcel? If not, what is the relative humidity of the air parcel at that pressure level?

Prob 2. (60\%) In the afternoon of a hot summer day, the surface generally warms up (by solar radiative heating) faster than the air aloft. This produces a near-surface layer with an enhanced lapse rate of temperature, as illustrated in Fig. 1. Consider an environmental temperature profile, shown as the solid line in Fig. 1, with a constant lapse rate of $8^{\circ} \mathrm{C} / \mathrm{km}$ above 300 m and an enhanced (also constant) lapse rate of $11^{\circ} \mathrm{C} / \mathrm{km}$ from the surface to 300 m height. The surface temperature and pressure are given as $300^{\circ} \mathrm{K}$ and 1000 hPa . The air is completely dry. If an air parcel located at the surface is pushed up with an initial upward velocity of $0.01 \mathrm{~m} / \mathrm{s}$ (i.e., $w(0)=0.01 \mathrm{~m} / \mathrm{s}$ and $\mathrm{z}(0)=0$, where $w(\mathrm{t})$ and $\mathrm{z}(\mathrm{t})$ are the vertical velocity and height of the parcel), find the maximum height, $H$, that the parcel will reach. (This is the height where the parcel comes to a stop.) In addition, plot the vertical velocity of the parcel as a function of height over the range of $0 \leq \mathrm{z} \leq H$. [As usual, we assume that the parcel undergoes an adiabatic process (i.e., it does not exchange heat with the environment) as it ascends. We also neglect friction between the parcel and the environment.]


Prob 3. (60\%) Solve Prob 11, Part (a) and (b), in Chapter 4 of the textbook. In Part (a), in addition to determining the specific humidity at $\mathrm{z}=3 \mathrm{~km}$, please also make a plot of the vertical profile of specific humidity, $\mathrm{q}(\mathrm{z})$, over the entire cloud column from 1 km to 9 km . (Note: Depending on your approach, the given information of " $p_{T}=330 \mathrm{mbar}$ " might be redundant.)

Prob 5. (15\%) Solve Prob 12 in Chapter 4 of the textbook. Attached below is a plot of the observed long-term climatology of the vertical velocity in p-coordinate at the 500 hPa level over Africa. Is your estimate of the vertical velocity over Sahara from Part (c) of that problem consistent with observation?

Note: The quantity shown in the figure below is the vertical velocity in p-coordinate, $\omega$ ( $\equiv \mathrm{dp} / \mathrm{dt}$ ), instead of the vertical velocity in z-coordinate, $w$. As indicated in the plot, it has the unit of $\mathrm{Pa} / \mathrm{s}$. The two vertical velocities are approximately related to each other by $\omega \approx-\rho \mathrm{g} w$. Note that a positive value of $\omega$ means downward motion. In fact, vertical velocity is not directly observed. The so-called observed vertical velocity, as shown, is derived from the vertical integration of horizontal divergence (based on the observed horizontal velocity at multiple pressure levels) by the continuity equation. The plot was constructed with the online analysis tool at NOAA Earth Systems Research Lab, Physical Science Division website (http://www.esrl.noaa.gov/psd), using the "NACP/NCAR Reanalysis data".


