MAE 578, Spring 2017 Homework #3

This assignment will contribute to 10% of the total score for the semester. Collaboration is not allowed.

1. (35%) In the vicinity of the City of Tempe, marked by a circle in Fig. 1, a rectangular grid of meteorological stations was set up to collect observations of temperature, velocity, etc. The spacing of the stations is 30 km in both x- and y-direction. At 9 PM of a certain day, the observation of the flow at the pressure level of z = 3 km indicates that (see illustration in Fig. 1) **(i)** The horizonal velocity is uniform in space and points to the northeast with a speed of 5 m/s. The wind vectors (red arrows) form a 45° angle with the x-axis, and **(ii)** The temperature contours (bold gray lines) are straight and equally spaced lines that form an acute angle, θ, with the x-axis where tan θ = 0.5. The temperature at Tempe is 9°C. At the particular location marked by a circle in Fig. 1, additional observations made at the z = 2.5 km and z = 3.5 km levels reveal that **(iii)** the temperatures there are T(2.5 km) = 14.5°C and T(3.5 km) = 5.5°C.

From the observations of the horizontal velocity at other vertical levels, someone has performed a vertical integration of the horizontal wind divergence to determine that **(iv)** the vertical velocity at z = 3 km is approximately –5 cm/s (a subsidence) uniformly over the area of our interest. Lastly, someone has also performed a radiative transfer calculation to estimate that **(v)** The radiative <u>cooling</u> rate ($\dot{Q}/c_p$ is negative) of the atmosphere at z = 3 km is approximately 2°C/hour uniformly over the area of our interest. Using the information from **(i)**-**(v)** and ignore the effect of moisture, try to make a prediction of the temperature at z = 3 km for Tempe at 10 PM (i.e., an hour later) using the standard temperature equation,

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + (\Gamma - \Gamma_d)w + \dot{Q}/c_p \ , \qquad \text{Eq. (1)}$$

where $\Gamma \equiv -\partial T/\partial z$ is the vertical lapse rate of the environmental temperature and $\Gamma_d \equiv g/c_p$ is the dry adiabatic lapse rate. To simplify the calculation, you may further assume that **(vi)** the velocity field remains the same over the one hour period from 9 PM - 10 PM.

Note: Most weather observation and forecast systems use pressure (instead of height) as the vertical coordinate. For the purpose of this exercise this is not critical. We assume that the flow field shown in Fig. 1 is at a constant-z level and the "w" in Eq. (1) is the vertical velocity in z-coordinate.

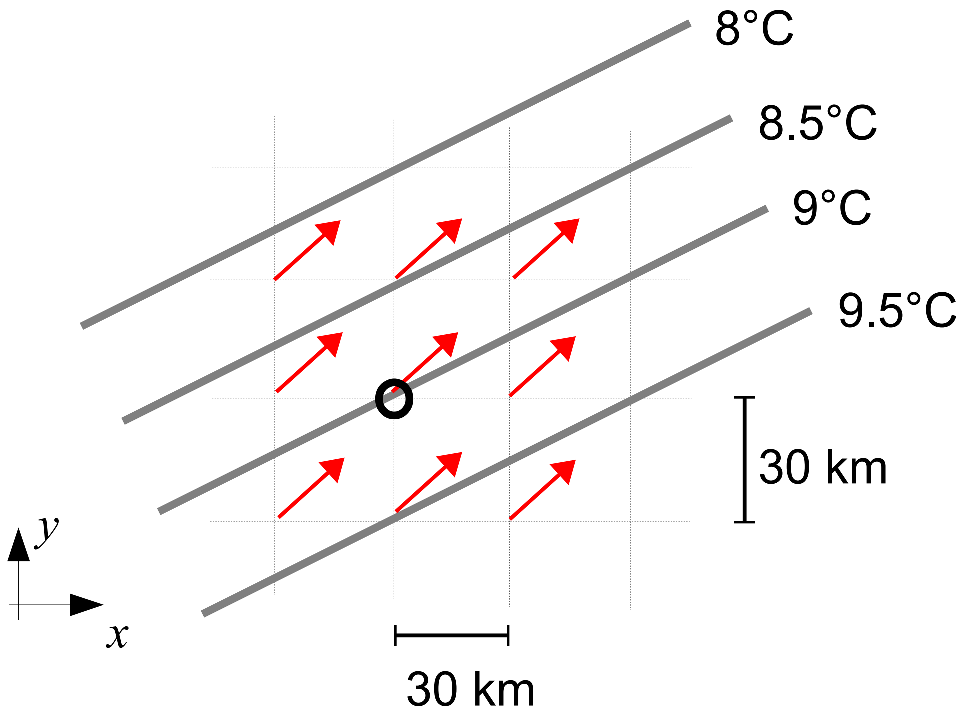

Fig. 1

2. (50%) Ignore vertical motion and consider the "2-D" horizontal motion of an air parcel in the x-y plane. Suppose that there is an externally maintained pressure gradient in the y-direction with $(\partial p/\partial y)$ = constant, while $(\partial p/\partial x) = 0$, across the domain. In this exercise, we want to calculate the trajectory of an air parcel with a given initial velocity. Consider the three cases (i)-(iii). In each case, three calculations will be performed with the initial velocity given as (u = 1 m/s, v = 0), (u = 3 m/s, v = 0), and (u = 0, v = –3 m/s).

Case (i): The parcel is initially located at 30°N. The imposed pressure gradient is $(\partial p/\partial y) = 0.1$ Pa/km.

Case (ii): The parcel is initially located at 30°N. The imposed pressure gradient is $(\partial p/\partial y) = -0.1$ Pa/km.

Case (iii): The parcel is initially located at 30°S. The imposed pressure gradient is $(\partial p/\partial y) = 0.1$ Pa/km.

Please make three plots separately for each case. In each plot, please collect the three trajectories together. For plotting purpose, you can set the initial location as (x, y) = (0, 0) for all cases. For each trajectory, integrate the equation of motion to 12 hours. You may assume that the density of the air is a constant, $\rho = 1$ kg/m^3. Since within 12 hours the parcel does not travel across a wide range of latitude, you may also approximate the Coriolis parameter, $f = 2\Omega \sin\phi$, as a constant taking its value as that at the initial latitude of the air parcel. Briefly discuss the results. [For example, you will find that in Case (ii), when the initial velocity is (u = 1 m/s, v = 0) the parcel drifts northward (in addition to moving eastward as expected from the initial velocity), while when the initial velocity is (u = 3 m/s, v = 0) the air parcel drifts southward. Explain why.]

3. (5%) Solve Part a of Prob 3 in Chapter 6 of the textbook. You do not need to solve Part b of that problem.

4. (5%) Solve Prob 4 in Chapter 6 of the textbook.

5. (5%) Solve Prob 5 in Chapter 6 of the textbook. You can ignore the last question, "What analogies can you draw ...".