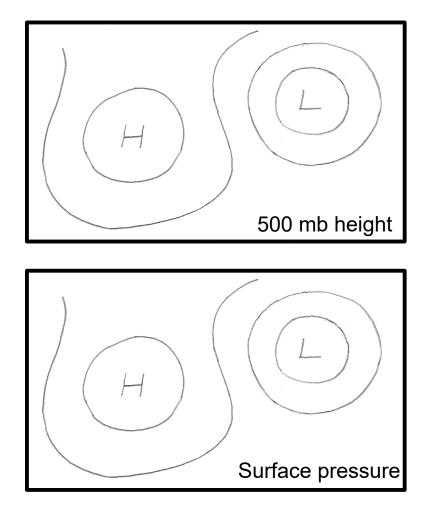
## MAE578, Spring 2017 Homework #5

This assignment is equivalent to one and a half regular homework. It will contribute to 15% of the total score for the semester. Collaboration is not allowed.

**Prob 1**. (20%) The following are two hypothetical maps of 500 mb height (top) and surface pressure (bottom, assuming that the surface is flat), each for a large-scale flow over a region in the *Southern Hemisphere*. Try to draw the anticipated horizontal velocity vectors, in the fashion of Figs. 7.4 and 7.25, given the pressure/height pattern. For the surface map (draw the flow near the surface), you should consider the effect of friction. Provide a brief explanation of your drawing (for example, by sketching the balance of forces for a selected wind vector).



**Prob 2**. (20%) Solve Prob 1(a) in Chapter 8 of the textbook. You do not need to solve Prob 1(b) and 1(c).

**Prob 3**. (30%) Solve Prob 2 in Chapter 8. Qualitatively sketch the trajectory of an air parcel that starts at the upper troposphere at 10°S and moves to 20°N while preserving its absolute angular momentum.

## Prob 4. (40%) Solve Prob 5 in Chapter 8.

[**Remarks on Prob 4**: It is useful to note that the "temperature surface" in Fig. 8.17 is essentially the "density surface". Then, the application of the idea from Sec 4.2 is straightforward. The problem has been simplified by assuming an incompressible flow with  $\rho = \rho(T)$ , which is not true for the atmosphere. To prepare for Prob 5, recall that for the atmosphere (which is close to an ideal gas and is vertically compressible) the "density" in the argument about buoyancy and convection should be replaced by potential temperature. The "density surface" in Fig. 8.17 should be replaced by the isentropic surface, and so on. From Prob 4, we note that "slantwise convection" can grow only when the slope of the "slantwise path" of the air parcel (*s* in Fig. 8.17) is shallower than the slope of the isentropic surfaces (*s*<sub>1</sub> in Fig. 8.17). This implies a "(horizontal) short-wave cutoff" in that a disturbance which is not shallow enough cannot grow.]

**Prob 5**. (40%) In long-term observation, there exists approximately a 40°C difference in surface air temperature between the equator and the pole which is maintained by differential solar heating. (See Fig. 5.7) Since temperature also generally decreases with height, in the latitude-height plane (as in Fig. 5.7) the isothermal surfaces (along which temperature is constant) "slope down" towards the pole. In contrast, the isentropic surfaces (along which potential temperature or entropy is constant) "slope up" towards the pole (see Fig. 5.8) because potential temperature increases with height in a statically stable atmosphere.

(a) Assume that surface air temperature drops from 25°C at the equator to -15°C at the pole. Further assume that sea level pressure is 1000 hPa, independent of latitude, and that the atmosphere is, on the average, statically stable with  $\Gamma \equiv -\partial T/\partial z = 8 \text{ C}^\circ/\text{km}$ . Estimate the slope of the isentropic surface in midlatitudes in the latitude-height plane. Does your estimate agree with the observed slope in Fig. 5.8? [Note that Figs 5.7 and 5.8 are actually in latitude-pressure plane but the relations relevant to this problem can be easily converted to their counterparts in the latitude-height plane. At the end, we require that the final answer of this problem be given in the latitude-height plane. In other words, the "slope" should be a non-dimensional quantity.]

(b) Repeat (a) but now assume that  $\Gamma = 9 \text{ C}^{\circ}/\text{km}$ . Does the slope of isentropic surfaces increase or decrease when the atmosphere becomes statically less stable (when all other parameters are kept the same)? In the context of the "short-wave cutoff" mentioned in **Remarks on Prob 4**, which atmosphere ( $\Gamma = 8 \text{ C}^{\circ}/\text{km} \text{ vs}$ .  $\Gamma = 9 \text{ C}^{\circ}/\text{km}$ ) will restrict midlatitude storms to a shorter meridional (north-south) length scale?