**Atmospheric "greenhouse effect"**
- How the presence of an atmosphere makes Earth's surface warmer

Some relevant parameters and facts (see previous slide sets)

$(S_o/4) \approx 342$ W m$^{-2}$ is the average incoming solar radiative energy per unit area for planet Earth.
($S_o \approx 1368$ W m$^{-2}$ is the Solar constant)

$\alpha_p$ is planetary albedo. In the absence of an atmosphere, it would be the surface albedo.

Stefan-Boltzmann law: A body with temperature T emits radiation with the energy per unit area = $\sigma T^4$
$\sigma \approx 5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ is Stefan-Boltzmann constant; $T$ in °K

Earth's atmosphere is nearly transparent to solar (shortwave) radiation
The "greenhouse gases" in the atmosphere do absorb terrestrial (longwave) radiation

Important greenhouse gases: $H_2O$ , $CO_2$ , $CH_4$ , etc.

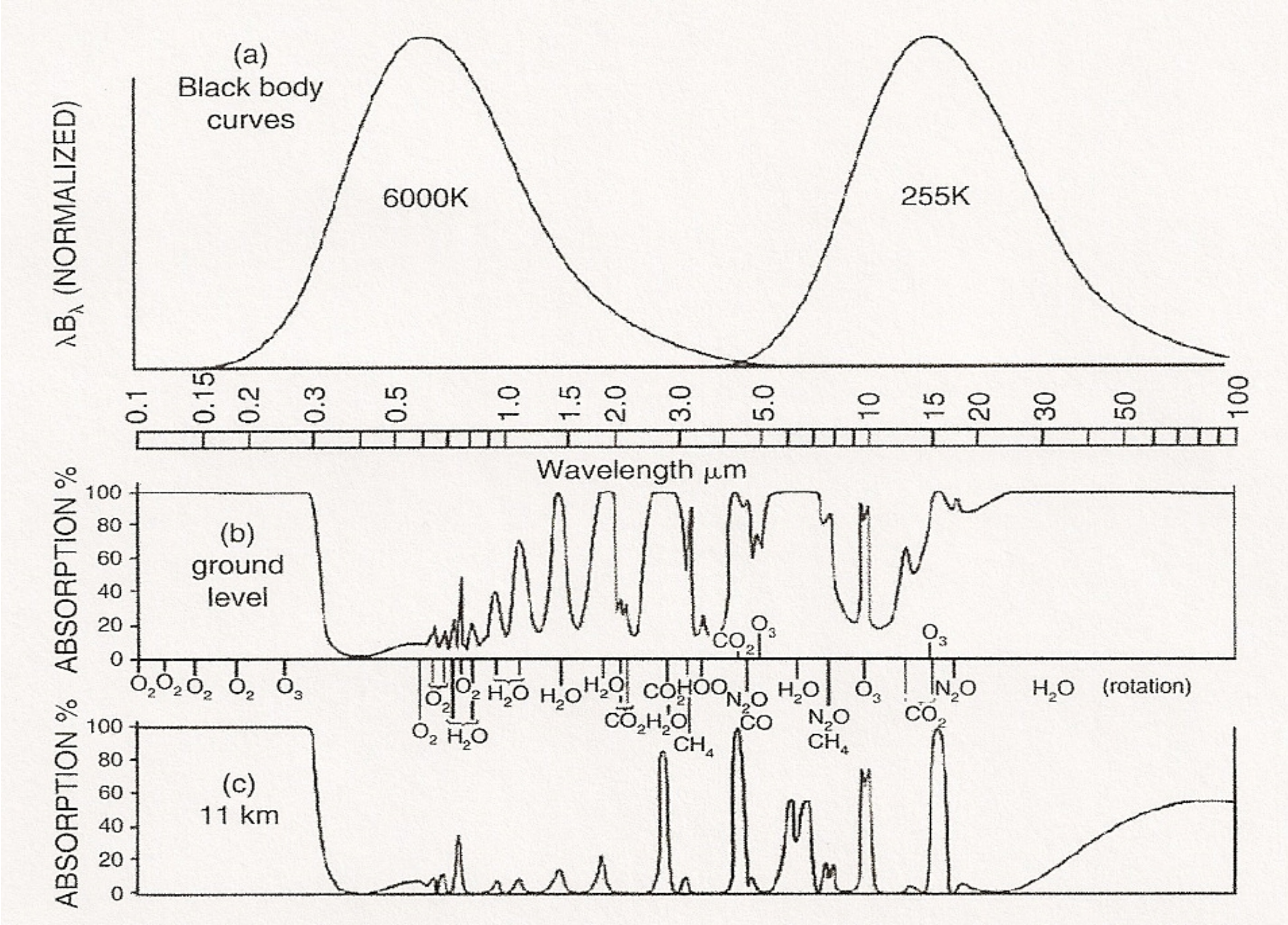



Fig. 2.6 in M&P (yet again)

**Case 1. Earth in radiative equilibrium without an atmosphere**

The incoming solar energy per unit area is $(S_o/4)\ (1 - \alpha_p)$

At temperature $T_E$, the energy per unit area radiated by the Earth is (by Stefan-Boltzmann law) $\sigma\ T_E^4$

Let "in = out", $\sigma\ T_E^4 = (S_o/4)\ (1 - \alpha_p)$ , we have

$$T_E = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$$

For example, if $\alpha_p = 0.1$, we have $T_E \approx 271.4$ K

Notations (for convenience of discussion):

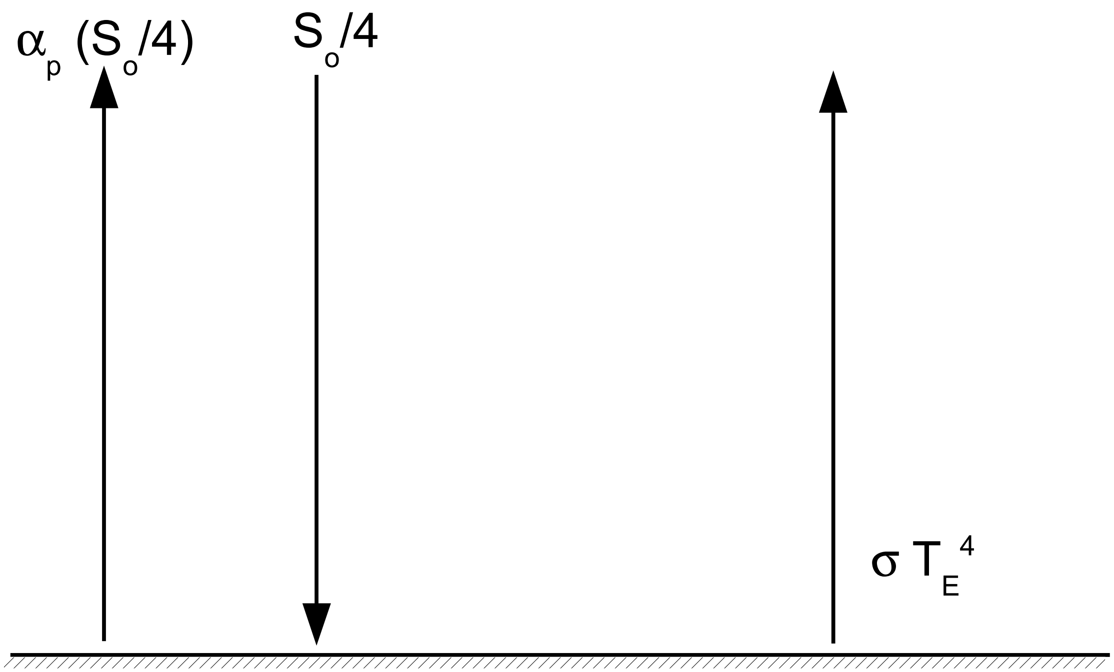
SW : "shortwave radiation" : the radiation directly emitted by the Sun, or the sunlight reflected by the Earth system), $\lambda \sim$ visible band

LW: "longwave radiation" : the radiation emitted by the Earth and/or its atmosphere, $\lambda \sim$ infrared band

SW $\uparrow$ : Upward shortwave radiation

LW $\downarrow$ : Downward longwave radiation, etc.

In previous example (earth with no atmosphere):

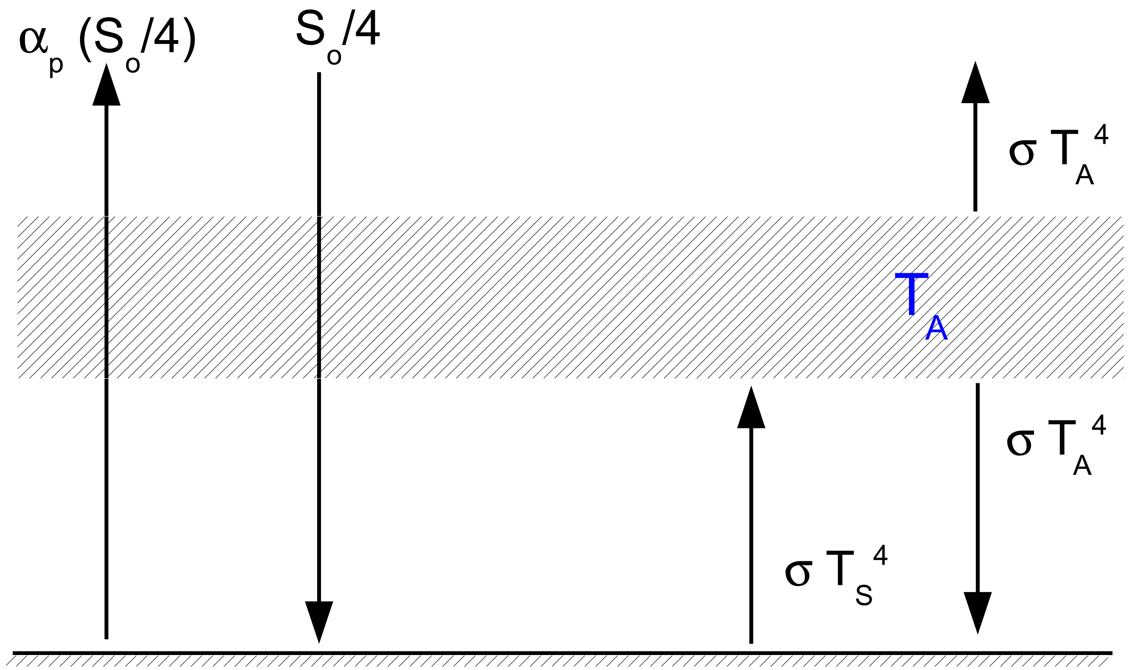



At the surface, SW ↓ = $(1-\alpha_p)(S_o/4)$ , LW ↑ = $\sigma T_E^4$

Radiative equilibrium @ surface: **SW ↓ = LW ↑**

=> $T_E = [(S_o/4)(1 - \alpha_p)/\sigma]^{1/4}$

Case 2. One-layer atmosphere that is transparent to SW but completely absorbing of LW radiation




@ surface: $SW\downarrow = (1-\alpha_p)(S_o/4)$ , $LW\downarrow = \sigma T_A^4$ , $LW\uparrow = \sigma T_S^4$

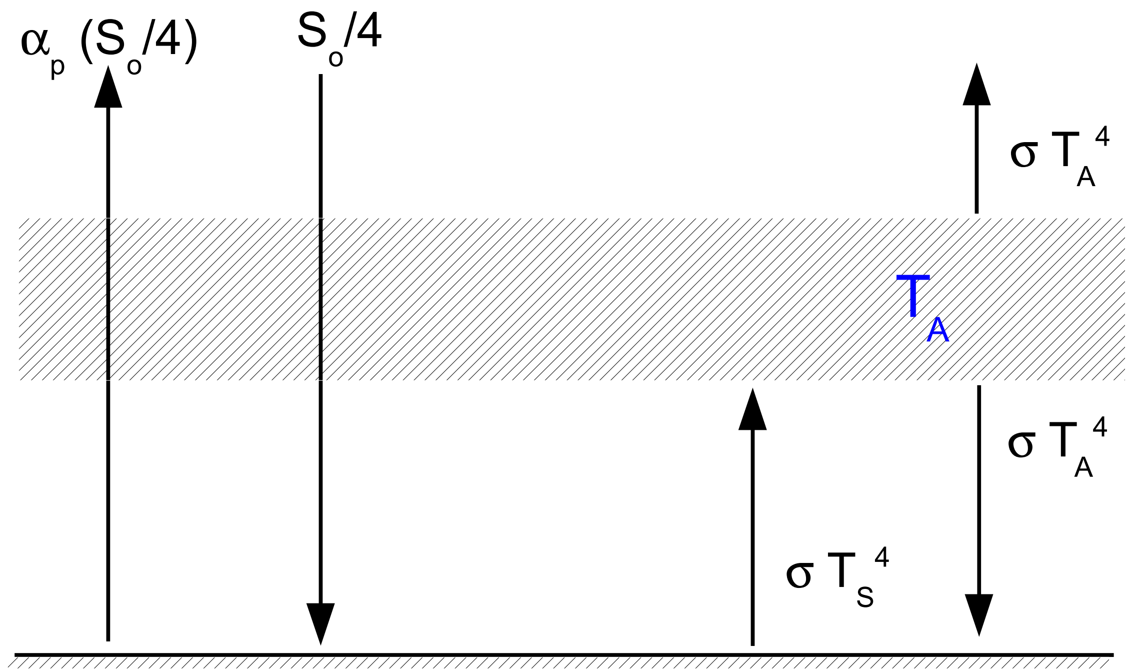
Radiative equilibrium for Earth surface: $\mathbf{SW\downarrow + LW\downarrow = LW\uparrow}$

=> $\sigma T_S^4 = (S_o/4)(1 - \alpha_p) + \sigma T_A^4$ Eq. (1)

@ top of atmosphere: $SW\downarrow = (1-\alpha_p)(S_o/4)$ , $LW\uparrow = \sigma T_A^4$

Radiative equilibrium for the earth system: $\mathbf{SW\downarrow = LW\uparrow}$

=> $\sigma T_A^4 = (S_o/4)(1 - \alpha_p)$ Eq. (2)




$\sigma T_S^4 = (S_o/4)(1 - \alpha_p) + \sigma T_A^4$ Eq. (1)

$\sigma T_A^4 = (S_o/4)(1 - \alpha_p)$ Eq. (2)

=> $T_A = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$

and

$2 \sigma T_A^4 = \sigma T_S^4$ => $T_S = 2^{1/4} T_A$

For example, if $\alpha_p = 0.1$, we have $T_A \approx 271.4$ K and $T_S \approx 322.7$ K (surface is warmer)

Note :

(1) The $T_A$ is now the effective temperature, $T_E$, for the Earth system (concerning its emission of radiation back to space). But note that it remains the same as the $T_E$ in the absence of an atmosphere

=> The presence of greenhouse gases in the atmosphere makes the surface of the Earth warmer, but it does not modify the total (Earth surface + atmosphere) longwave radiation that the Earth system emits back to space, i.e. LW↑ at the top of the atmosphere. In radiative equilibrium, this quantity depends only on the Solar constant and planetary albedo.

(2) $T_S > T_A$ is always true, regardless of the detail of the greenhouse-gas absorptivity, etc.

# The case with partial absorption and emission of IR radiation

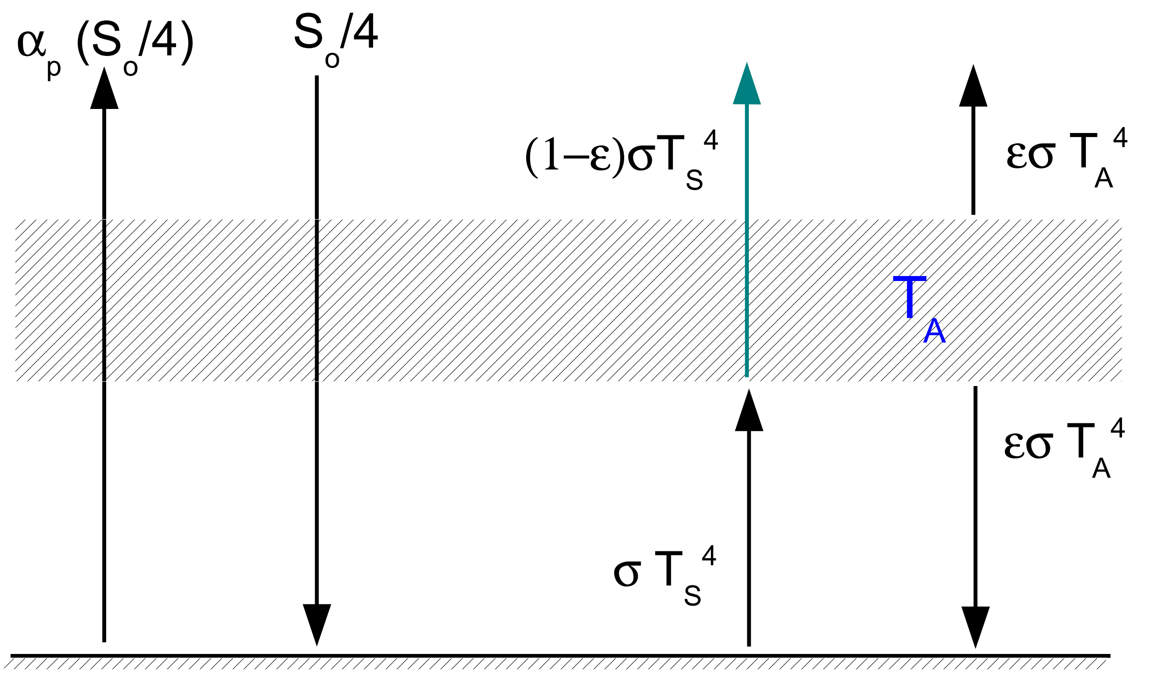
At local thermodynamic equilibrium, Kirchoff's law states that

*absorptivity* = *emissivity*

If an object has a temperature $T$ (in °K) and an emissivity of $\varepsilon$, it will emit radiation of $\varepsilon \, \sigma T^4$

Note that $0 \le \varepsilon \le 1$

Case 3. One-layer atmosphere that is transparent to SW and partially absorbing of LW radiation, with $\varepsilon$ being the fraction of IR absorption by the atmosphere




@ surface: $SW\downarrow = (1-\alpha_p)\ (S_o/4)$, $LW\downarrow = \varepsilon\ \sigma\ T_A^4$, $LW\uparrow = \sigma\ T_S^4$

=> $\sigma\ T_S^4 = (S_o/4)(1 - \alpha_p) + \varepsilon\ \sigma\ T_A^4$ Eq. (1)

@ top of atmosphere: $SW\downarrow = (1-\alpha_p)\ (S_o/4)$, $LW\uparrow = \varepsilon\ \sigma\ T_A^4 + (1-\varepsilon)\sigma\ T_S^4$

=> $\varepsilon\ \sigma\ T_A^4 + (1-\varepsilon)\sigma\ T_S^4 = (S_o/4)(1 - \alpha_p)$ Eq. (2)

(1) & (2) => $T_S = [2/(2-\varepsilon)]^{1/4}\ T_E$ , $T_A = [1/(2-\varepsilon)]^{1/4}\ T_E$

where $T_E = [(S_o/4)(1 - \alpha_p)/\sigma]^{1/4}$

Summary of the results of a "one-layer atmosphere" model

| Temperature | No atmosphere | Partial IR absorption ($\varepsilon < 1$) | 100% IR absorption |
|---|---|---|---|
| surface, $T_S$ | $T_E$ | $[2/(2-\varepsilon)]^{1/4}\ T_E$ | $2^{1/4}\ T_E$ |
| atmosphere, $T_A$ | -- | $[1/(2-\varepsilon)]^{1/4}\ T_E$ | $T_E$ |

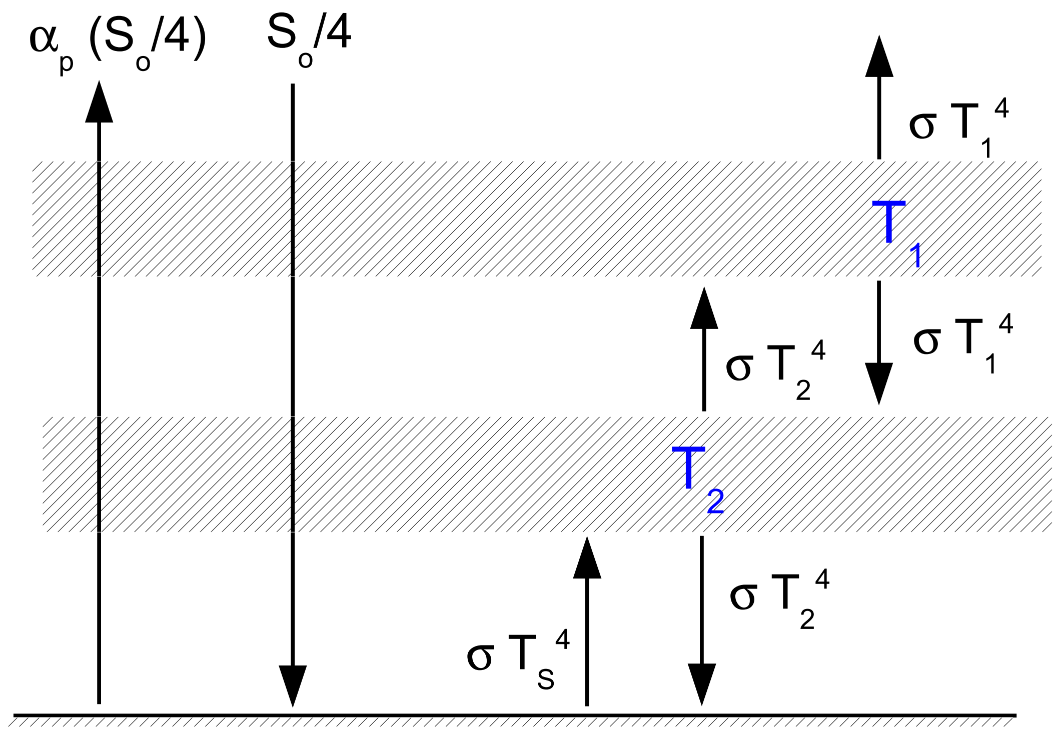
$T_E = [(S_o/4)(1 - \alpha_p)/\sigma]^{1/4}$ is the temperature of Earth in the absence of an atmosphere.

For example, for $S_0/4 = 342$ W m$^{-2}$, $\alpha_p = 0.1$, $\varepsilon = 0.8$, we have $T_E = 271.4$ K, $T_A = 259.2$ K, and $T_S = 308.3$ K

$T_S > T_A$ always holds. The atmosphere receives IR energy from the surface. Therefore, it cannot be warmer than the surface. In a multi-layer model, we will see that temperature decreases away from the surface.

Smaller $\varepsilon$ => Colder atmosphere. This is expected, since the atmosphere relies on the intake of energy, by absorbing IR radiation, to keep itself warm. (We have already assumed that the atmosphere is transparent to SW radiation, therefore LW radiation is the only energy source for the atmosphere.)

Case 4. Two layers of atmosphere, both with 100% IR absorptivity (but remain transparent to SW radiation) $T_E = T_1$ in this case




@ TOA, $\sigma T_1^4 = (S_o/4)(1 - \alpha_p)$ $T_1 = [(1/\sigma)(S_o/4)(1 - \alpha_p)]^{1/4}$ Eq. (1)

@ Surface, $\sigma T_S^4 = \sigma T_2^4 + (S_o/4)(1 - \alpha_p) = \sigma T_1^4 + \sigma T_2^4$ Eq. (2)

For layer 2, $\sigma T_1^4 + \sigma T_S^4 = 2\,\sigma T_2^4$ Eq. (3)

Combine (1)-(3): $T_S = 3^{1/4}\, T_1$ , $T_2 = 2^{1/4}\, T_1$ => $\mathbf{T_S > T_2 > T_1}$

**Temperature decreases with height**

**Case 5**

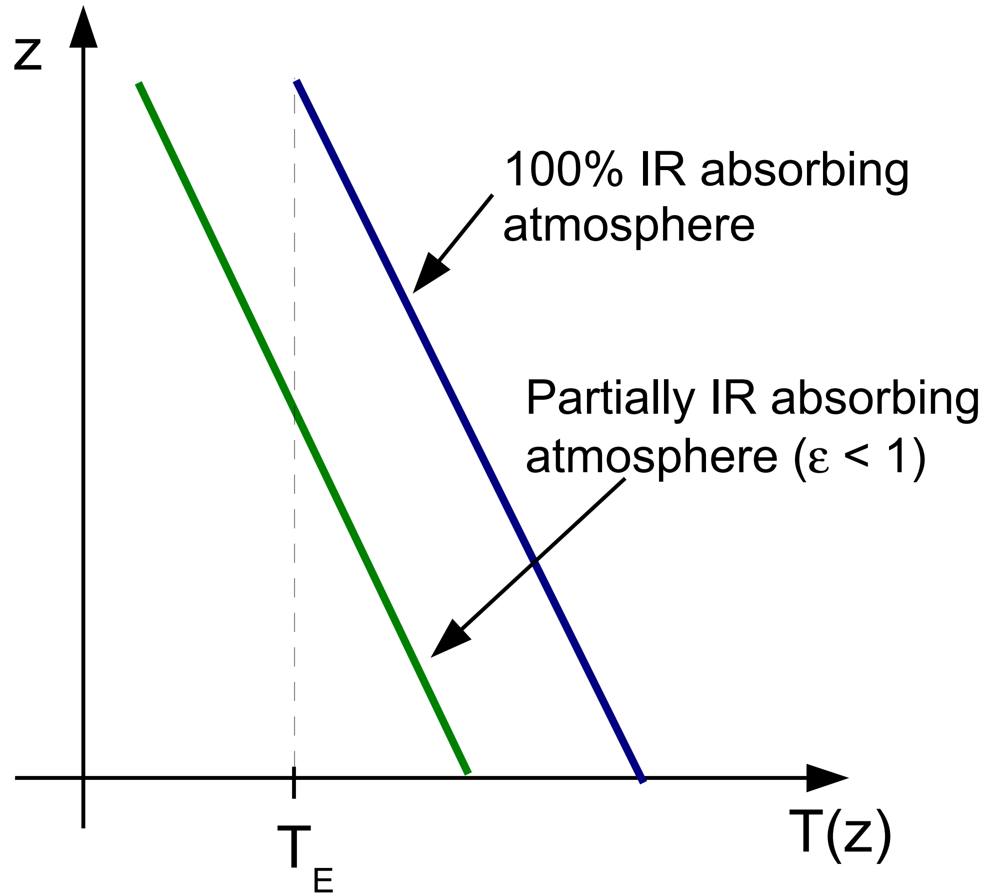
If we modify Case 4 by adding more layers, the conclusion will remain the same that temperature decreases with height, while the top layer always has the temperature of $T_E = [(S_o/4)(1 - \alpha_p)/\sigma]^{1/4}$.

In this case, the surface becomes warmer if we add more (totally absorbing) layers. In other words, as the atmosphere becomes thicker (having more greenhouse gases in it - only greenhouse gases count), the surface of the planet becomes warmer (cf. Prob 6 of Chap 2 in M&P textbook).

**Case 6**

If we modify Case 4 or 5 by assuming partial IR absorption ($\varepsilon < 1$) for the atmospheric layers, the resulted temperature will still decrease with height (surface being the warmest). Overall, the atmosphere and the surface become warmer with an increasing $\varepsilon$.

# Summary (radiative equilibrium temperature profile)



Radiative energy balance determines the "first order" picture of the vertical temperature distribution. Recall that radiative transfer is but one of the three ways to move heat around. Heat conduction, and especially convective heat transfer by atmospheric motion, will act to modify this baseline temperature profile. Vertical convection will happen when the radiatively maintained temperature profile becomes statically unstable.