MAE578, Spring 2017 Term project

Computation of thermal convection in stratified flow
(Comparison of hydrostatic, Boussinesq, anelastic, and fully non-hydrostatic models)

Background

In our discussion on the governing equations for environmental fluid dynamics, it was noted that most of the numerical models used in real applications are either hydrostatic or fully non-hydrostatic. A hydrostatic model does not allow a direct computation of vertical acceleration but simply "diagnoses" vertical velocity from mass continuity. The fully non-hydrostatic model admits acoustic waves whose existence forces us to use a very small time step size, Δt, to maintain numerical stability.

Between the hydrostatic and fully non-hydrostatic versions of the "primitive equations", there exist other systems with intermediate levels of complexity that are more accurate than the hydrostatic model (e.g., the vertical component of the momentum equation is made to be prognostic instead of diagnostic) but still exclude sound waves. We briefly discussed Boussinesq approximation in Chapter 4. Another example is the *anelastic* equation (Ogura and Phillips 1962). The purpose of this project is to survey and compare different systems for the computation of thermal convection in stratified flows.

Main tasks

Read the original and review papers listed below on Boussinesq, anelastic, and related approximations and perform a literature survey to find more articles on the subject. From the literature survey,

(1) Compare the structures of the Boussinesq, anelastic, and related systems and discuss how they differ from the hydrostatic and fully non-hydrostatic systems. Describe how the momentum, thermodynamic, and continuity equations in those systems differ from each other.

(2) Describe the conditions under which the anelastic approximation is valid. Provide examples of phenomena which can be accurately simulated by an anelastic model.

(3) Explain how acoustic waves are suppressed in the Boussinesq and anelastic systems. Note that the generation of sound waves is related to compressibility of the fluid, quantified by the change in density in response to the change in pressure, ($\partial \rho / \partial p$).

(4) Survey the literature to find examples of published results of numerical simulations that used the anelastic equation. Investigate why the anelastic system has not been more widely used in practice (compared to hydrostaic and fully non-hydrostatic systems).

References

The three papers listed below are the starting points of the required literature survey. You are encouraged to further expand the list to enhance the discussions on the key points listed under *Main tasks*.

Ogura, Y., and N. A. Phillips, 1962: Scale analysis of deep and shallow convection in the atmosphere, *J. Atmos. Sci*., **19**, 173-179

Lilly, D. K., 1996: A comparison of incompressible, anelastic, and Boussinesq dynamics, *Atmos. Res*., **40**, 143-151

Durran, D. R., and A. Arakawa, 2007: Generalizing the Boussinesq approximation to stratified compressible flow, *C. R. Mecanique*, **335**, 655-664

In addition to the three main references, the following reports (one for a hydrostatic global model, the other for a non-hydrostatic regional model) provide some detail of the numerical architecture of hydrostatic and non-hydrostatic models:

Kiehl, J.T., J.J. Hack, G.B. Bonan, et al., 1996: Description of the NCAR Community Climate Model (CCM3). NCAR Technical Note NCAR/TN-420+STR, DOI: 10.5065/D6FF3Q99.

Skamarock, W., J.B. Klemp, J. Dudhia, et al., 2008: A Description of the Advanced Research WRF Version 3. NCAR Technical Note NCAR/TN-475+STR, DOI: 10.5065/D68S4MVH.

All of these papers and reports are publicly available. (The papers are available through ASU library online.) Please contact the instructor if you have difficulty obtaining the papers..

The suggested length of the term paper for this project is 15-20 pages, including figures, tables, and references. Reports that are slightly longer or shorter are acceptable, as long as they are of good quality.