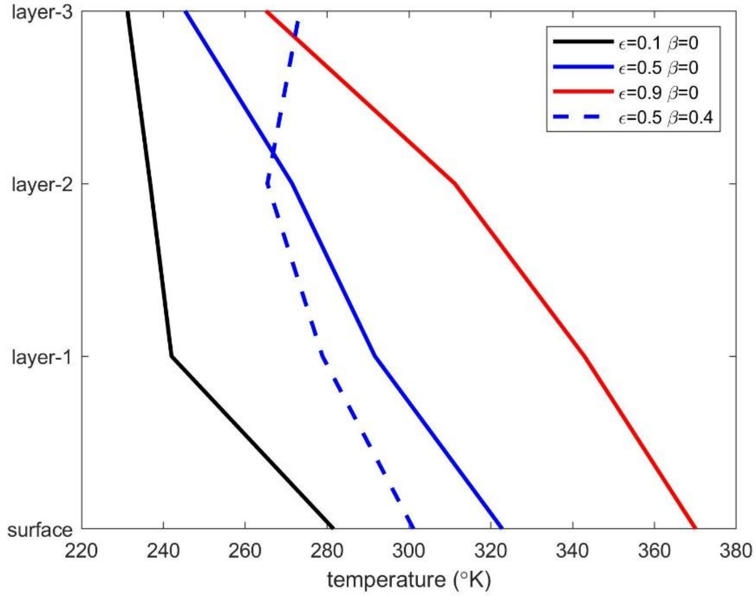
MAE578 Spring 2019 HW1 Solution

Prob 1

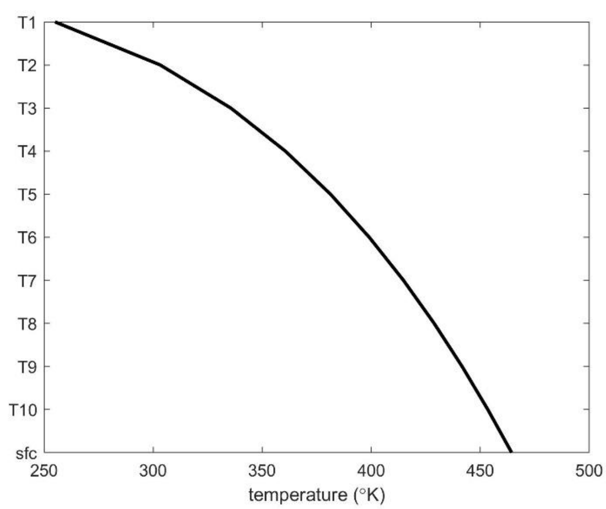


Surface temperature:

Case 1: 281.57 °K, Case 2: 322.79 °K, Case 3: 370.05 °K, Case 4: 301.09 °K

Prob 2

Part (c) As a byproduct of proving the results in Part (a) and (b), one can show that $T_n = n^{1/4}\, T_1$. Combined with $T_1 = T_e$ and $T_s = (n+1)^{1/4}\, T_e$, this determines the temperature profile for the case with N = 10 as shown below. (We use T_e = 255 K.)



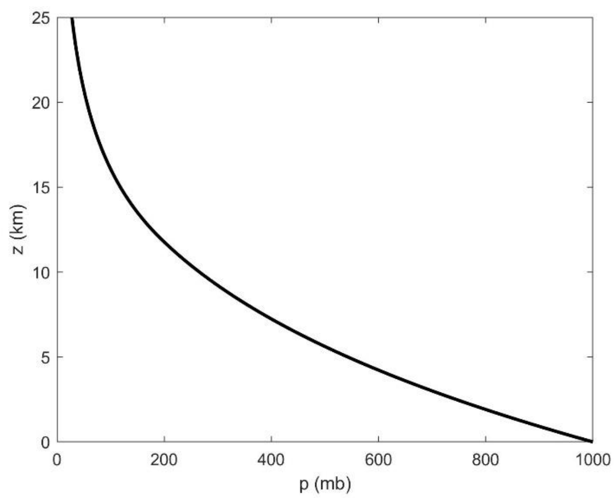
Prob 3

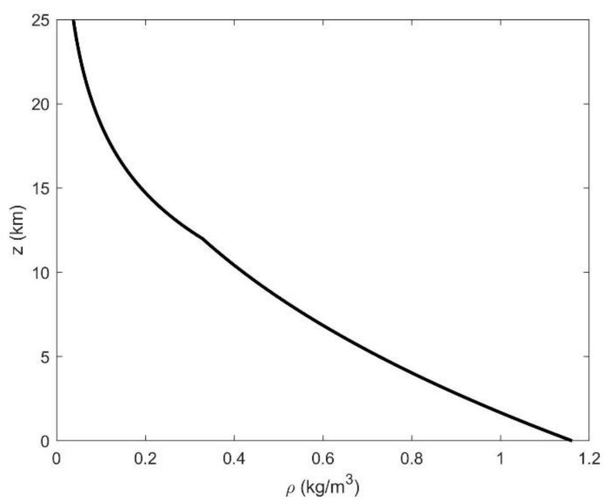
(a)

$$p(z) = \begin{cases} p(0)\left[\dfrac{T_1(z)}{T(0)}\right]^{-\frac{g}{R\alpha}} \;, \; if\ z\ \le 12\ km \\ p(0)\left[\dfrac{T_1(12km)}{T(0)}\right]^{-\frac{g}{R\alpha}}\left[\dfrac{T_2(z)}{T_1(12km)}\right]^{-\frac{g}{R\beta}} \;, \; if\ 12km < z\ \le 25\ km \end{cases}$$

From p(z), density ρ(z) can be computed from ideal gas law, $\rho(z) = p(z)/(R\ T(z))$.

p(12 km) = 192.4 mb p(25 km) = 27.6 mb ρ(12 km) = 0.328 kg/m^3 ρ(25 km) = 0.037 kg/m^3





(b) Exact answer:

$$\Delta Z = -\frac{1}{\alpha}T(0)[p(0)]^{\frac{R\alpha}{g}}\left[(80000)^{-\frac{R\alpha}{g}} - (30000)^{-\frac{R\alpha}{g}}\right] = 7300.5\ m$$