## MAE 578, Spring 2019 Homework \#2

One set of homework $\approx 10 \%$ of the total score for the semester. Hard copy of report is due at 4:35 PM on the due date. Please follow the rules for collaboration as described in the cover page of Homework \#1. Computer codes should be included in the report. A statement on collaboration is required for all reports, including those that are done independently.

For all 3 problems, the atmosphere is an ideal gas with gas constant $R=287 \mathrm{~J} \mathrm{~kg}^{-1}{ }^{\circ} \mathrm{K}^{-1}$ and specific heat $c_{p}=(7 / 2) R=1005 \mathrm{~J} \mathrm{~kg}^{-1}{ }^{\circ} \mathrm{K}^{-1}$. If needed, other relevant thermodynamic properties of dry air can be found in Table 1.4 of M\&P textbook. The atmosphere is in hydrostatic balance with $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$.

Prob 1 (65\%)
An observation (by conventional sounding) of temperature as a function of pressure was made at a meteorological station. The text file, "mae578_2019_hw2_data.txt", contains the data. A matlab code for reading the data is given in the Appendix. The code and data are available at our class website. The first record in the data ( $p=924 \mathrm{mb}, T=12.2^{\circ} \mathrm{C}$ ) is the measurement taken at the surface. The elevation of the station is 751 m . (At the surface, the height is already $\mathrm{z}=751 \mathrm{~m}$.)
(a) From the data, calculate and plot the vertical profiles of pressure ( $p$ ), temperature $(T)$, density $(\rho)$, and potential temperature ( $\theta$, using $p_{\mathrm{S}}=924 \mathrm{mb}$ as the reference pressure) as a function of height. If tropopause is defined as the location with the minimum of temperature, what is the height of the tropopause for this observed profile?
(b) Calculate and plot the vertical profile of the lapse rate of temperature, $\Gamma \equiv-\mathrm{d} T / \mathrm{d} z$, as a function of height. Compare it to the dry adiabatic lapse rate, $\Gamma_{\mathrm{d}} \equiv g / c_{p}$, and use this as the basis to discuss the (dry) static stability of the atmospheric column. (Your result here should be consistent with the plot of $\theta(z)$ from Part (a). Note that the stability criterion, $\Gamma<\Gamma_{\mathrm{d}}$, is equivalent to $\mathrm{d} \theta / \mathrm{d} z>0$.)

Prob 2 (10\%)
From an observation at a certain location we are informed that the density of the atmosphere is constant over a vertical segment (say $z_{1} \leq z \leq z_{2}$ ). In other words, density is independent of height. Is the atmospheric profile within that segment statically stable, unstable, or neutral? Also, determine the lapse rate, $\Gamma \equiv-\mathrm{d} T / \mathrm{d} z$, of the environmental temperature profile within that segment.

Prob 3 (25\%)
Solve Prob 12 in Chapter 4 of the textbook. Figure 1 shows a plot of the observed long-term climatology of the "vertical velocity in $p$-coordinate" at the 500 mb level over Africa. Is your estimate of the vertical velocity over Sahara from Part (c) of that problem consistent with observation?

Note: The quantity shown in Fig. 1 is $\omega(\equiv \mathrm{d} p / \mathrm{d} t)$ instead of the vertical velocity in z-coordinate, $w$. (As indicated in the figure, $\omega$ has the unit of $\mathrm{Pa} / \mathrm{s}$.) The two are approximately related by $\omega \approx-\rho g \omega$, and the density of atmosphere at the 500 mb level is approximately $0.7 \mathrm{~kg} / \mathrm{m}^{3}$. Note that a positive value of $\omega$ means downward motion. In fact, vertical velocity is not directly observed. The so-called observed vertical velocity, as shown, is derived from the vertical integration of horizontal divergence (based on the observed horizontal velocity at multiple pressure levels) by the continuity equation. The figure was generated using the online analysis tool at NOAA Earth Systems Research Lab, Physical Science Division website (http://www.esrl.noaa.gov/psd), based on the "NACP/NCAR Reanalysis data".


Fig. 1

Appendix: How to read the data for Prob 1
The data file, "mae578_2019_hw2_data.txt", is in plain text and consists of two columns for pressure (in mb ) and temperature (in ${ }^{\circ} \mathrm{C}$ ). The following matlab code, also available at our class website, can be used to read the data from the file into two arrays, $p$ and $T$, each with 115 elements. The first pair of records, $(p(1), T(1))$, are the pressure and temperature at the lowest elevation (or highest pressure), which is the surface. As a quick reference, the matlab code also plots $T$ as a function of $p$, as shown below.

```
clear
fid1 = fopen('mae578_2019_hw2_data.txt','r');
for n = 1:115
    p1 = fscanf(fid1,'%f7.1');
    T1 = fscanf(fid1,'of7.1');
    p(n) = p1; T(n) = T1;
end
plot(T,P,'b-','LineWidth',2)
axis([-80 20 0 1000])
xlabel('T(p) (degC)'); ylabel('p (hPa)')
set(gca,'YDir','reverse')
```



