MAE578, Spring 2019 Homework #6
See HW1,2 for rules on collaboration and guideline on submission of report

Prob 1. (15%) Figure 1 shows two hypothetical contour maps of the geopotential height field at (a) a level above the top of planetary boundary layer, and (b) a level near the surface, within the boundary layer (assuming that the surface is flat). Assume that the two maps depict a large-scale system in the mid-latitude of the *Southern Hemisphere*. Moreover, assume that geostrophic balance holds at the level above boundary layer. Try to draw the anticipated horizontal velocity vectors (by superimposing them on the two maps in Fig. 1), in the fashion of Fig. 7.4, 7.24, or 7.25, given the patterns of the height field. For the map in Fig. 1b, you should consider the effect of friction. Provide a brief explanation of your drawing (for example, by sketching the balance of forces at a selected location where you draw a wind vector, in the fashion of Fig. 7.22).

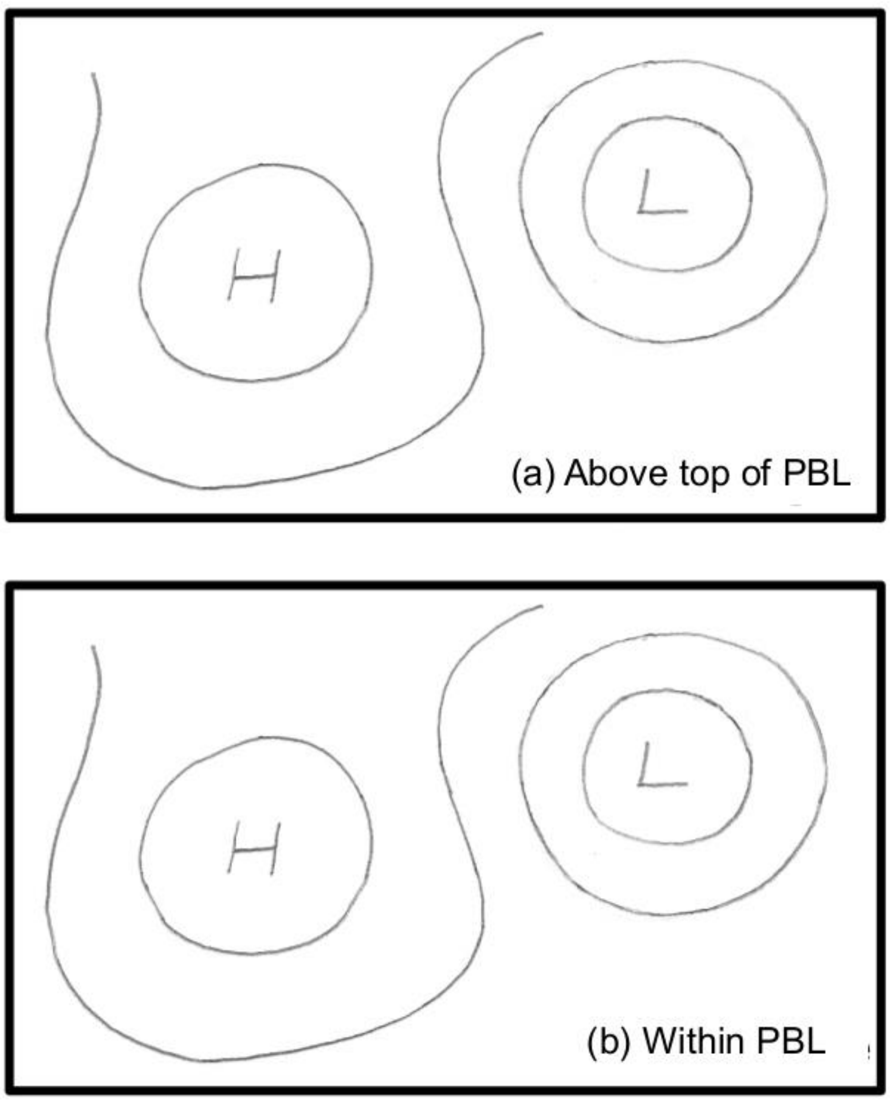

Fig. 1

Prob 2 (10%)
Consider a steady (horizontal) 2-D axially-symmetric circulation (or a "vortex") in the Northern Hemisphere with a low-pressure center (see Fig. 2a). According to the "three-way balance" discussed in class, in polar coordinate the balanced momentum equation in the radial direction can be written as

$$0 = \frac{V^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r} + fV$$

where V is the tangential velocity (defined as positive if the flow is counterclockwise). The three terms are the inertial term (in the form of a "centrifugal acceleration"), pressure gradient force, and Coriolis force. The three forces are denoted as Ce, PGF, and Co in Fig. 1. For the steady axially-symmetric flow, the inertial term is essentially a "centrifugal acceleration" that always points outward in the radial direction (i.e., $V^2/r > 0$, regardless of the sign of V). Given the pressure minimum at center, PGF points inward ($\partial p/\partial r > 0$, therefore PGF $= -\rho^{-1}\partial p/\partial r < 0$). For a "normal" type of balanced flow in the Northern Hemisphere associated to a low-pressure center, we have a cyclonic (counterclockwise) circulation with positive V. This type of 3-way balance is illustrated in Fig. 2a. For this system, Rossby number (Ro) can be written as $V/(f\,r)$, where r is the radius of the vortex at which the balance of forces is considered. When $Ro << 1$, $V^2/r << fV$ such that the 3-way balance is reduced to a 2-way balance (i.e., geostrophic balance) with $V \approx V_g$, where $V_g \equiv (f\rho)^{-1}\partial p/\partial r$ is geostrophic wind. If V is the velocity obtained from the full 3-way balance (necessary when Ro is not small), write the ratio, V/V_g, as a function of Ro. Plot V/V_g vs. Ro over the range of $0 \leq Ro \leq 1$. In the context of the Final Project, what is the relative residual for geostrophic balance when $Ro = 1$?

Prob 3 (20%)
Consider a 2-D axially symmetric circulation in the Northern Hemisphere with a high-pressure center, as shown in Fig. 2b. In this case, the normal "three-way balance" is as depicted in Fig. 2b: PGF and Ce point outward while Co points inward. The tangential velocity, V, is negative (i.e., the flow goes clockwise). Otherwise, the equation given in Prob 2 is still valid. Given the magnitude of the pressure change from the edge (at radius r) to the center of the vortex, $\Delta p = p(r) - p(0)$, we could estimate the PGF as approximately $-(\Delta p/\rho)/r$. Then, the equation for the three-way balance can be approximated as

$$0 = \frac{V^2}{r} - \frac{(\Delta p/\rho)}{r} + fV$$

(Note that while this equation supports two solutions, one of them is unphysical as it implies that the magnitude of V would keep increasing with r. This solution should be ignored, and the "normal" solution be used for the rest of this problem.) First, show that when $\Delta p < 0$ (i.e., the system has a high-pressure center) there is a minimum cutoff of r, r_{cut}, such that the radius of the balanced axially-symmetric circulation cannot be smaller than r_{cut}. (This restriction only applies to a high-pressure system. A low-pressure system can have $r < r_{cut}$.) Given $\Delta p = -5$ mb ($= -500$ Pa), $\rho = 1$ kg/m^3, and $f = 10^{-4}$ s^{-1}, calculate r_{cut}. (Please express it in km.) Under these given parameters, solve V from the three-way balance equation and plot V vs. r over the range of $r_{cut} \leq r \leq 3r_{cut}$.

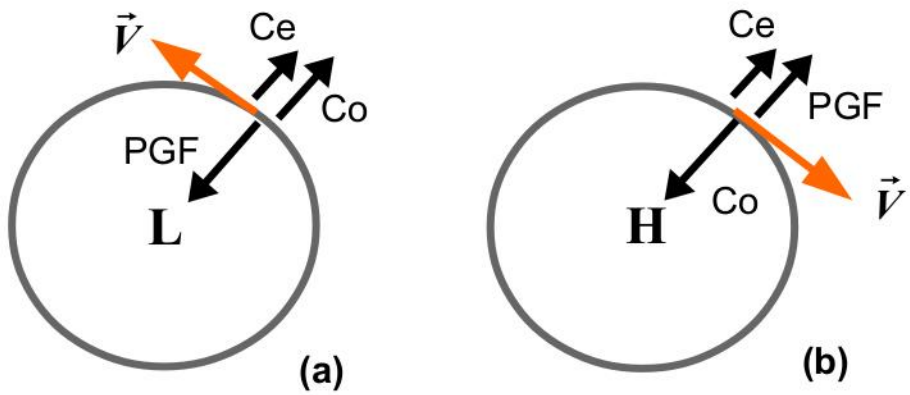

Fig. 2

Prob 4. (15%) Solve Prob 2 in Chapter 8. Qualitatively sketch the trajectory of an air parcel that starts at the upper troposphere at 10°S and moves to 20°N while preserving its absolute angular momentum.

Prob 5. (40%) Solve Prob 5 in Chapter 8.

[Remarks on Prob 5: It is useful to note that the "temperature surface" in Fig. 8.17 is essentially the "density surface". Then, the application of the idea from Sec 4.2 is straightforward. The problem has been simplified by assuming an incompressible flow with $\rho = \rho(T)$, which is generally not true for the atmosphere. Recall that for the atmosphere (which is close to an ideal gas and is vertically compressible) the "density" in the argument about buoyancy and convection should be replaced by potential temperature. The "density surface" in Fig. 8.17 should be replaced by the isentropic surface, and so on. From the solution of this problem, we will learn that "slantwise convection" can grow only when the slope of the "slantwise path" of the air parcel (s in Fig. 8.17) is shallower than the slope of the isentropic surfaces (s_1 in Fig. 8.17). This implies a "(horizontal) short-wave cutoff" in that a disturbance which is not shallow enough cannot grow by the so-called "baroclinic instability".]