

MAE578 Spring 2019 Term Project
Geostrophic and thermal wind balance: Theory and observation

1. Background

The second half of this course focuses on the dynamics of large-scale flows under the influence of Earth's rotation (Coriolis effect). Analyzing the governing equations of fluid motion in a rotating frame, we find that large-scale flows approximately follow the so-called "geostrophic balance" (Eq. 7-3, 7-4, or 7-8 in M&P textbook) that relates velocity to pressure (or height) gradient, or "thermal wind balance" (Eq. 7-18 or 7-24) that relates vertical wind shear to horizontal temperature gradient. Through these *balance relations* we understand (among many other things) why, in the Northern Hemisphere, a low-pressure center on a weather map is associated with a "cyclonic vortex" with a counterclockwise flow pattern. Given the importance of the balance relations, the purpose of this project is to analyze observational data to determine how accurately these relations hold for the real atmosphere.

In p -coordinate, geostrophic balance can be written as

$$\mathbf{V} \approx \frac{1}{f} \mathbf{k} \times \nabla \Phi$$

where \mathbf{V} is the horizontal wind vector, f is Coriolis parameter, \mathbf{k} is the local vertical (upward) unit vector, and $\Phi = gz$ is geopotential. For this project, the relation can be recast as

$$\mathbf{V} = \frac{1}{f} \mathbf{k} \times \nabla \Phi + \text{Residual}$$

In p -coordinate, the thermal wind relation can be expressed as (Eq. 7-24)

$$\frac{\partial \mathbf{V}}{\partial \ln(p)} \approx -\frac{R}{f} \mathbf{k} \times \nabla T$$

Or, it can be recast as

$$\frac{\partial \mathbf{V}}{\partial \ln(p)} = -\frac{R}{f} \mathbf{k} \times \nabla T + \text{Residual}$$

The key task of this project is to quantify the residual as a function of latitude, height, and other parameters (see *Suggested deliverables*) using an observational data set. To analyze the accuracy of geostrophic balance or thermal wind balance, it is more meaningful to consider the relative residual, i.e., residual as a percentage of either of the main terms in the balance relation. Through this exercise, one will also become more familiar with the structures of large-scale wind, temperature, and pressure (or height) fields, and their inter-relations.

2. *The reanalysis data*

The "observational data" for the required calculations will come from one of the so-called "reanalysis" data sets. They typically have a global coverage and 6-hourly temporal resolution over the span of several decades. (The raw meteorological observations are spatially and temporally inhomogeneous. They have been interpolated and quality-checked as part of the process of creating the reanalysis data on regular grids.) The reanalysis data for 3-D temperature, geopotential height, and horizontal wind fields are archived in pressure coordinate. This makes it more convenient to carry out the analysis of residuals using the equations in p -coordinate.

The data are archived at several public (free access) websites. We recommend the web portal of NOAA Earth System Research Lab/Physical Science Division: www.esrl.noaa.gov/psd. The website is user-friendly and is equipped with online tools that allow users to assemble and download subsets of data for a specific field with a specific type (daily, monthly, etc.) of time averaging. A brief tutorial will be given by the instructor on how to navigate the website. For the purpose of this project, either NCEP-NCAR Reanalysis I or NCEP-DOE Reanalysis II will work. The data are archived in netCDF format and can be read by calling Matlab built-in functions. A note on using Matlab to process netCDF data will be provided separately. More background on the two reanalysis data sets can be found in the following references

Kalnay, E., et al., 1996: The NCEP/NCAR reanalysis project, *Bulletin of the American Meteorological Society*, vol. **77**, 437-470

Kanamitsu, M., et al., 2002: NCEP-DOE AMIP Reanalysis (R2), *Bulletin of the American Meteorological Society*, vol. **83**, 1631-1643

Yet more background information about reanalysis can be found at the website, reanalyses.org.

3. *Guideline for the final report*

The expected length of the final report is 15-20 pages, including figures, codes, and references. The report should include the numerical methods used for the key calculations. (For example, for the thermal wind relation, describe how the horizontal temperature gradient and vertical wind shear are computed from the 3-D grided data and compared at the same location.) To ensure reproducibility, the main results should be accompanied by a description of the specific data used in the calculations. For example, each calculation of the residual should be accompanied by the information of time/date/month/year and location (both horizontal location and vertical level). In the report, it is also essential to include discussions on how and why the residual varies with latitude, height, and geographical location (e.g., over sea vs. land), etc.

4. *Suggested deliverables*

The computation carried out in this project should help illustrate the conditions under which

geostrophic or thermal wind balance hold. Comparisons should be made between two cases with contrasting conditions (e.g., high latitudes vs. tropics). Note that the balance relations generally hold under small Rossby number, but several factors determine the Rossby number for a flow in a particular region and at a particular time. We suggest the following points for potential comparisons:

(1) Compare the accuracy (quantified by the relative residual) of geostrophic and/or thermal wind balance as a function of latitude. Since the Coriolis terms diminish towards the Equator, qualitatively we expect the balance relations to break down in lower latitudes.

(2) Compare the accuracy of geostrophic and/or thermal wind balance as a function of height. This is concerned with the effect of surface friction on the balance relations. Since the effect of friction diminishes outside the planetary boundary layer (from the surface to ~ 1 km height), this comparison is meaningful only if one chooses two locations such that one is at a height sufficiently above the top of boundary layer (e.g., at 500 mb level) and the other is within the boundary layer (e.g., at 925 mb level). A useful way to examine the deviation from geostrophic balance is to plot the horizontal wind vector and check whether there is a significant cross-isobar component of it. (If the flow is parallel to the isobars, it is in good geostrophic balance.)

Note: In the "pressure-level data" in reanalysis, if a pressure level is below the ground, the wind and temperature at that level is artificially extrapolated downward onto that level. At many locations, the wind and temperature at 1000 mb and 925 mb are not real. For a meaningful comparison for (2), make sure that at a chosen location the pressure levels used for the comparison are above ground.

(3) Compare the accuracy of geostrophic and/or thermal wind balance as a function of overall wind speed. Under a high-wind condition, the inertial term in momentum equations may become comparable to Coriolis term, and Rossby number would approach $O(1)$. This is the case with a three-way balance among the inertial, pressure gradient, and Coriolis terms, and the relative residual for geostrophic or thermal wind balance may be large.

(4) Since the Coriolis parameter has opposite signs in the Northern and Southern Hemisphere, it would also be interesting to show the contrast in the overall picture of geostrophic or thermal wind balance between the two hemispheres.

Expectation: While it is not mandatory to perform all of (1)-(4), we expect that at least two out of (1)-(3) are included in the report. (Point (4) is relatively minor.) For example, a satisfactory report could consist of more in-depth analyses for (1) and (2), or shorter analyses for (1), (2), and (3). Also, it is not necessary to check both geostrophic balance and thermal-wind balance for each of (1)-(4), but in the report there should be at least one set of calculation for each balance relation. For example, if (1) and (2) are the main focus of a report, it is acceptable to analyze only thermal wind balance for (1), and only geostrophic balance for (2).