MAE578 Homework 2 (Solar & terrestrial radiation)

"Lunar constant" *(3 points)*

1. (A) Adopting the same procedure and definitions used to derive the Solar constant (S_o = 1368 W m^{-2}), try to estimate its lunar counterpart, the "Lunar constant" L_o, that represents the radiative energy (per unit cross-sectional area) of moonlight intercepted by the Earth. The surface albedo of the Moon is about 0.1. The radius of the Moon is 1700 km and the Moon-to-Earth distance is 380,000 km. Compare L_o to S_o. All computer models for weather and climate prediction ignore the effect of moonlight on Earth's radiative energy balance. Is it justified?
(B) During night time, how does the intensity of moonlight that reaches Earth's surface compare to that of the infrared radiation emitted by Earth's surface? Assume an average T_S = 240K for the Earth at night. In this context, does moonlight play a significant role in the radiation budget at Earth's surface during the night?

You may adopt these assumptions: (i) Ignore the infrared radiation emitted by the Moon but solely consider the effect of the second-hand sunlight reflected by the Moon that reaches the Earth. (ii) While the Moon-to-Sun distance varies with time, as an approximation we may fix it to 1 A.U. (iii) Assume that it's full moon all the time. This will lead to an overestimate of L_o, but we are happy to have an order-of-magnitude estimate. (iv) If the spherical geometry of the Moon's surface is too complicated for you, as an approximation try to treat the Moon as a flat disk (cf. Prob 2 of Chap. 2 in M&P textbook). You may make further assumptions if needed.

Greenhouse effect and vertical temperature profile *(3 points)*

2. (A) Work out Prob 5 of Chap.2 in M&P textbook. As an additional hint, note that since all atmospheric layers are "completely absorbing of IR radiation", the "planetary emission temperature" T_e is simply the temperature of the top layer, T_1 (since the infrared emission from lower layers cannot penetrate the top layer).
(B) Adopting a 10-layer model (N = 10) with $S_o/4$ = 342 W m^{-2} and α_p = 0.1, calculate the temperatures for all layers, T_n, n = 1, 2, ..., 10, along with the surface temperature, T_S. Plot the vertical temperature profile of the atmosphere using these 11 data points. Comment on the result.

Wien's displacement law *(1 point)*

3. Given Planck function $B_\lambda(T)$ in Eq. (A-1), for a fixed temperature T the peak wavelength of the radiative energy spectrum is where $\partial B_\lambda(T)/\partial\lambda = 0$. Use this to derive a formula of the peak wavelength λ^* as a function of T and show that λ^* decreases with an increasing temperature. (Thus, a star that looks blue is hotter than one that looks red.) What are the values of λ^*, in μm, for T = 4000K and T = 300K?