MAE578 Homework #3

Hand-drawn figures are **not acceptable** for Prob 2(b); Please use Matlab or other softwares.

Buoyancy oscillation

1. A tank is fill with a layer of oil (ρ = 0.8 g/cm^3) on top of a layer of water (ρ = 1.0 g/cm^3). A block of specially made plastic with ρ = 0.9 g/cm^3 and with a thickness of 10 cm is then carefully placed between the two layers. See Fig. 1. This system is statically stable. Starting from the perfectly balanced position with half of the block immersed in oil and half in water, if we push the plastic block down just slightly, the positive buoyancy it experiences will restore it back to its original position. Then, with its remnant upward velocity, it will keep moving upward until its kinetic energy is exhausted by negative buoyancy. The cycle would then continue as an oscillation of the vertical position of the block. *Estimate the period of this oscillation.* (3 points)

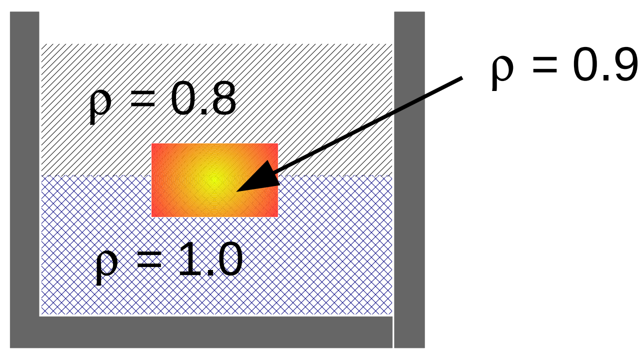


Fig. 1

Vertical structure and static stability

2. Assuming that between the surface and p = 100 mb the temperature (T) and pressure (p) of the atmosphere are related by

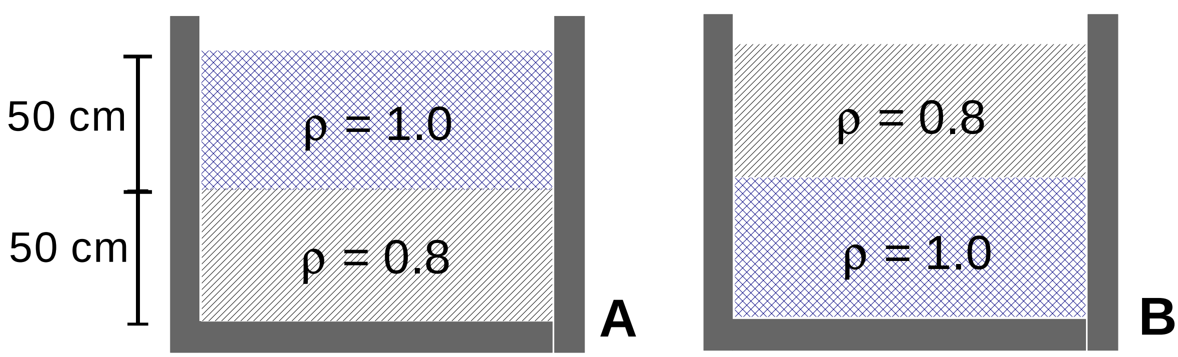
$$T(p) = A\,p^{B}\,, \qquad \text{Eq. (1)}$$

where T is in °K, p is in bar (1 bar = 1000 mb = 100000 Pa), A = 300 and B = 0.15. For instance, T = 300K at p = 1000 mb. Also, surface pressure is given as p_S = 1000 mb and surface elevation is z = 0. The atmosphere is in hydrostatic balance. (a) What is the height of the level with p = 100 mb? (b) Determine temperature, pressure, potential temperature (θ), and air density (ρ), as a function of height and plot T(z), p(z), θ(z), and ρ(z) from surface to the level with p = 100 mb. What are the values of T, p, θ, and ρ at z = 2 km? For the definition of θ, use p_S = 1000 mb as the reference pressure level. (c) Is this vertical profile statically stable? Determine the environmental lapse late, $\Gamma \equiv -(dT/dz)$ and compare it with the adiabatic lapse rate $\Gamma_d = g/c_p$. Also, find the critical value, B_C, such that (given Eq. (1)) a profile $T(p)$ with $B < B_C$ is statically stable while one with $B > B_C$ is unstable. (4 points)

Convection and energy conversion

3. A rectangular tank with a 1 m x 1 m base area is initially filled with a 50 cm layer of water (ρ = 1.0 g/cm^3) on top of a 50 cm layer of oil (= 0.8 g/cm^3), see state "A" in Fig. 2. This fluid system is statically unstable. A small perturbation would lead to overturning fluid motion (convection) that gains momentum when lighter fluid rises and heavier fluid sinks. This is a process of a conversion of gravitational potential energy to kinetic energy. Clearly, not all potential energy at the initial state can be converted to kinetic energy. The conversion process ceases when the fluid reaches a stable configuration of minimum potential energy, see state "B" in Fig. 2. The "available potential energy" (APE), i.e., the maximum amount of potential energy that can be converted to kinetic energy, is APE = $P_A - P_B$, where P_A and P_B are the total potential energy for state A and B. (a) Calculate the APE for the fluid system in state "A". (b) If all of the APE is converted to kinetic energy of the fluid flow, estimate the magnitude of the flow velocity. (3 points)

Hint: For Part (b), we will be satisfied with an order-of-magnitude estimate. It may greatly simplify the calculation if we assume that the kinetic energy gained from the PE-to-KE conversion is uniformly distributed in space.



(Base area = 1 m x 1 m)

Fig. 2