MAE578 Homework #5

Variation of temperature with latitude

1. Solve Prob 2 of Chap. 5 in M&P textbook. In addition, make a plot of surface temperature as a function of latitude. (2 points)

Static and total energy for the atmosphere

2. We have previously learned about the "moist static energy", $E_S \equiv C_P T + gz + Lq$ (see Sec. 4.5.2), which can be regarded as the energy (per unit mass) of an air parcel in the absence of any motion. Taking into account atmospheric motion, the total energy would be $E = E_S + E_K$, where $E_K \equiv (1/2)\,|\mathbf{v}|^2$ is kinetic energy ($\mathbf{v}$ is the 3-D velocity vector; cf. Eq. (8-14)). Using the collection of figures in Chap. 5 for the climatological mean state of the atmosphere as a function of latitude and height (or pressure), try to estimate the magnitude of the individual components of E at selected latitudes and pressure levels and fill the blanks in the following table. Comment on your results. The purpose of this exercise is for you to become familiar with the climatological state presented in Chap. 5. We will revisit the detail of energy balance in Chap. 8. (3 points)

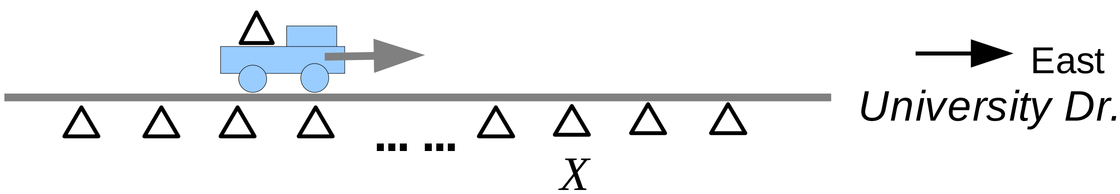
	$C_P T$	gz	Lq	$(1/2)\lvert\mathbf{v}\rvert^2$	E
@ Equator and 1000 mb level					
@ Equator and 200 mb level					
@ 45°N and 1000 mb level					
@ 45°N and 200 mb level					

All quantities are in m^2/s^2

Note: (1) The height field shown in Fig. 5.13 is the "anomaly", i.e., departure from a certain global-mean value. More precisely, if we denote the anomaly as $z^*(\phi,p)$ (ϕ is latitude) and the global mean as Z(p), then the total height is $z(\phi,p) = z^*(\phi,p)+Z(p)$. It is this total height that should be used for evaluating the "gz" term. For this exercise, let's assume that the global mean Z(p) is 12 km at p = 200 mb and 0 km at p = 1000 mb. (2) Strictly speaking, the $\mathbf{v}$ for the evaluation of kinetic energy should be the three-dimensional velocity. Since for global-scale circulation the "zonal component" (u) tends to be greater than v and w, for this exercise we will approximate $|\mathbf{v}|^2$ by $|u|^2$, where the magnitude of u can be inferred from its "zonal mean" in Fig. 5.20. (We have not discussed Fig. 5.20 but will come back to it after the introduction of thermal wind balance in Chap. 7.)

Eulerian vs. Lagrangian framework

3. A field campaign was conducted to assess the variation of temperature across the city of Mesa. Two teams were deployed to measure the air temperature along University Drive which runs in the east-west direction through the city. The first team consisted of local volunteers whose houses happen to be located on University Dr. Thermometers were set up in their front yards (illustrated as the line of triangles) for continuous monitoring of temperature at each location. The second team operated a mobile unit (see illustration) with a thermometer on board a truck that moved eastward along University Dr. at a speed of 5 m/s. The 1st team reported that at any given time during the campaign temperature decreases eastward along University Dr. with a constant rate of 0.1 °C/km. The 2nd team reported a constant decrease in temperature at a rate of 1.0 °C/hour as recorded by the mobile thermometer. What would be the rate of change (in time) of temperature as measured by any of the volunteers at a fixed location (for instance the one marked by an "X") ? (2 points)



Momentum, vorticity, and divergence

4. (a) Ignoring friction, if vertical velocity vanishes (w = 0) for a certain fluid flow, the horizontal components of the momentum equation can be written as

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial x} \quad , \qquad (1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y} \quad . \qquad (2)$$

If the density of the flow is uniform in the horizontal direction, show that Eqs. (1) and (2) lead to

$$\frac{d\zeta}{dt} = -\zeta D \quad , \qquad (3)$$

where $\zeta \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the *vorticity* and $D \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is the *divergence* of this flow. Therefore, if $D \equiv 0$ (flow is "non-divergent") we have *conservation of vorticity following the motion of a fluid parcel*. [Note: When w = 0 and ρ = constant, by the continuity equation D is guaranteed to be zero unless there is a mass source or sink.] (3 points)

(b) In the case when $D \neq 0$, Eq. (3) indicates that convergence ($D < 0$) leads to an amplification of the vortex motion while divergence ($D > 0$) leads to damping of vorticity. One can appreciate this behavior by momentarily holding D as a constant, which leads to $\zeta(t) = \zeta(0)\exp(-D\,t)$. This behavior is also consistent with daily experience; If we unplug a bathtub filled with water, the mass loss through the sinkhole would momentarily create a convergence ($D < 0$). Accompanying it, we see an amplification of the vortex motion surrounding the sinkhole. Now that we have a mathematical basis in Eq. (3), try to physically interpret this phenomenon. (1 point)