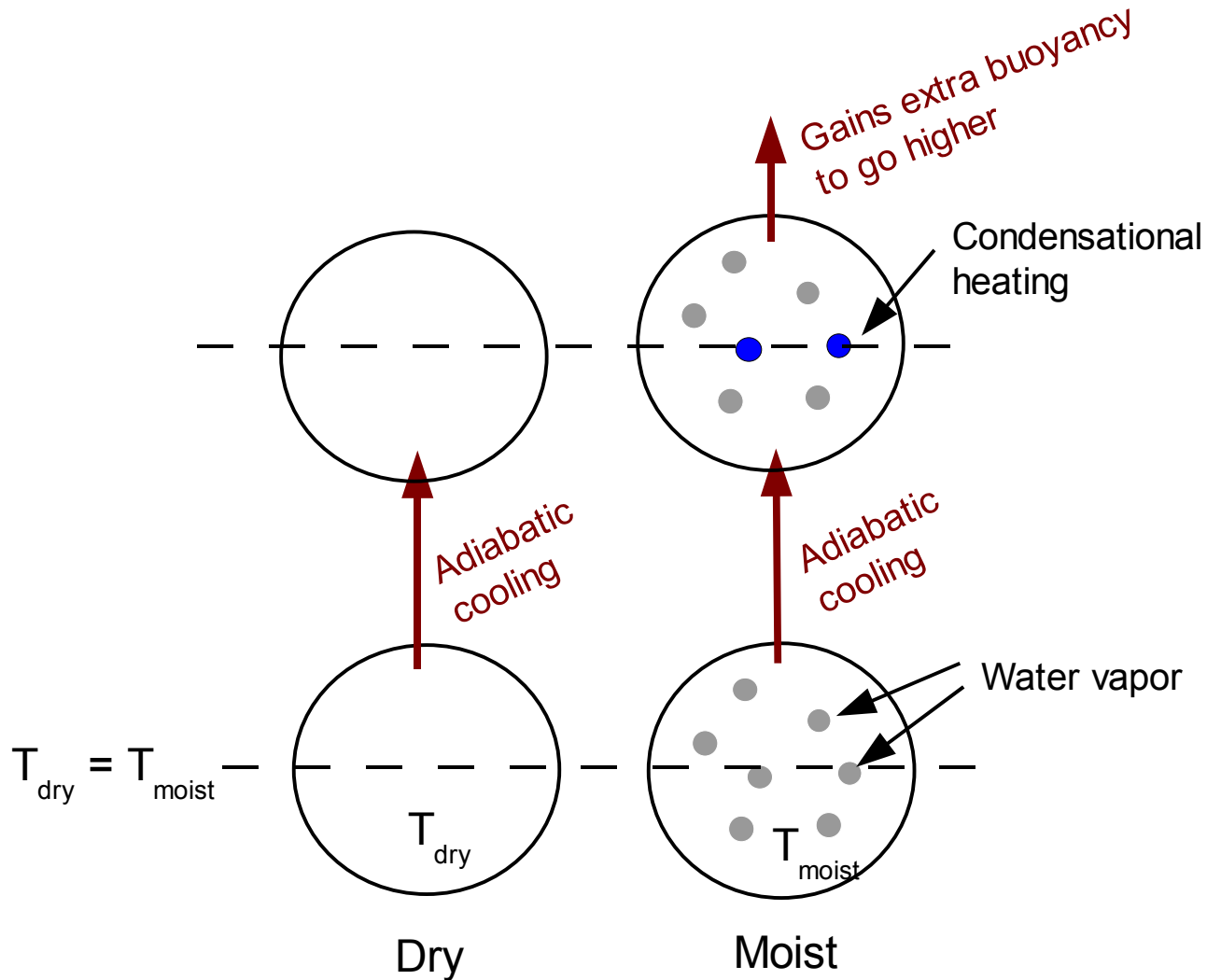


Effects of moisture on static stability & convection

Dry vs. "moist" air parcel: Lifting of an air parcel leads to adiabatic cooling. If the temperature of the parcel falls below the critical temperature for condensation, phase change may occur \Rightarrow Latent heat release \Rightarrow The "wet" air parcel gains extra buoyancy to ascend to a higher level. Thus, **moisture enhances convection**



Water vapor

Dynamics:

The effect of water vapor is minor before condensation occurs. What makes water vapor important is the effect of condensational heating in moist convection.

Radiation:

Water vapor is also a greenhouse gas that effectively absorb IR radiation emitted by Earth's surface.

Water is extremely important for human life. All of the water over land comes from the ocean. The transport of water vapor from sea to land, and the ensuing processes of moist convection and precipitation, are of first order importance in weather/climate prediction. To predict those processes, we need extra equations for water vapor and for phase change between water vapor and liquid water or ice. For example, the equation for the transport of water vapor by atmospheric motion would look like

$$\frac{\partial q}{\partial t} + \mathbf{v} \cdot \nabla q = \text{Source} + \text{Sink} ,$$

where \mathbf{v} and q are velocity and specific humidity. Condensation would count as a sink of water vapor (but a source of liquid water). We will revisit this point later.

Useful parameters for a moist atmosphere

Density

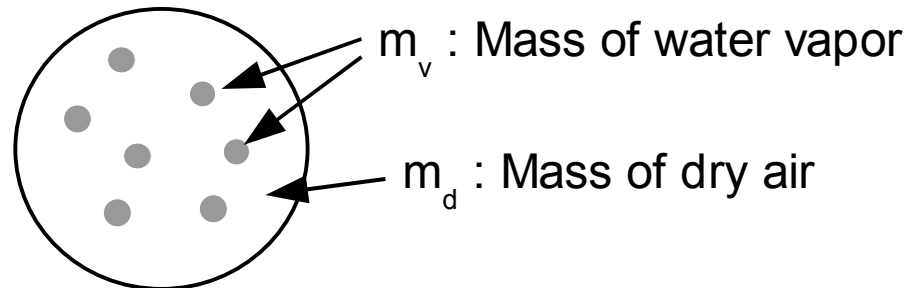
Consider an air parcel that contains water vapor

m_v : Mass of water vapor $\rho_v = m_v/V$ (V is the volume of the parcel)

m_d : Mass of dry air $\rho_d = m_d/V$

Total mass $m_{\text{tot}} = m_v + m_d$

\Rightarrow Total density $\rho = \rho_v + \rho_d$



Pressure

e : Partial pressure for water vapor

p_d : Partial pressure for dry air

Total pressure $p = e + p_d$ (Dalton's law)

Typically, $e \ll p$. At surface, e is ~ 30 mb at most, while $p \sim 1000$ mb.

Temperature is the same for water vapor, dry air, and the parcel as a whole

Ideal gas law is satisfied by the dry air and water vapor separately, with different gas constants

$$p_d = \rho_d R_d T$$

$$e = \rho_v R_v T$$

Gas constant

For many applications, we only need to know the ratio of R_d to R_v . (For the record, their values are $R_d \approx 287$ and $R_v \approx 461 \text{ J K}^{-1} \text{ kg}^{-1}$.) From basic chemistry,

$$R_d/R_v = M_v/M_d ,$$

where M_d and M_v are the molecular weight (not to be confused with the mass, m_d or m_v) of dry air and water vapor. Since dry air consists of $\sim 80\%$ nitrogen and $\sim 20\%$ oxygen,

$$M_d \approx 0.8 (\text{N}_2) + 0.2 (\text{O}_2) = 0.8 (28) + 0.2 (32) = 28.8 \text{ g/mol} ,$$

while $M_v = 1.0 (\text{H}_2\text{O}) = 18 \text{ g/mol}$. So we have

$$\varepsilon \equiv R_d/R_v = 18/28.8 \approx 0.62$$

We will use the constant ε to simplify later discussions.

Specific humidity

$$q \equiv m_v/m_{\text{tot}} = \rho_v/\rho ,$$

is the ratio of the mass of water vapor to the total mass (water vapor + dry air) of the air parcel. Note that q is dimensionless. The value of q is on the order of 0.01 (i.e., 10 g of water vapor for 1 kg of moist air.)

Mixing ratio (just for the record; we will not use it)

$$r \equiv m_v/m_d = \rho_v/\rho_d .$$

Relation between q and e

$$q = \frac{\rho_v}{(\rho_v + \rho_d)} = \frac{1}{1 + \left(\frac{\rho_d}{\rho_v}\right)} . \quad \text{Eq. (1)}$$

Combining the ideal gas law for dry air and water vapor, we have

$$\frac{p_d}{e} = \frac{\rho_d R_d}{\rho_v R_v} = \varepsilon \left(\frac{\rho_d}{\rho_v}\right) \quad \text{Eq. (2)}$$

Combining (1) and (2), and $p = e + p_d$, we have

$$q = \frac{1}{1 + \frac{1}{\varepsilon} \left(\frac{p}{e} - 1\right)} \quad \text{Eq. (3)}$$

Since $e \ll p$, one may take it as a good approximation that

$$q \approx \frac{\varepsilon e}{p} . \quad \text{Eq. (4)}$$

Key variables for a moist atmosphere

The state of a dry atmosphere can be described by the variable set, (p, T) , where p is pressure, T is temperature. Both are functions of space and time, (x, y, z, t) . For a moist atmosphere, we need (p, T, q) or (p, T, e) . Note that once (p, T, e) is known, q can be readily obtained from Eq. (3) or (4).

Saturation vapor pressure

The saturation vapor pressure, e_s , is the critical value of the partial pressure of water vapor such that condensation may occur when $e > e_s$. In other words, e_s indicates the maximum amount of water vapor (in terms of its pressure) that the atmosphere can accommodate under given pressure and temperature. As it turns out, e_s depends almost exclusively on temperature, as described by ***Clausius-Clapeyron equation***,

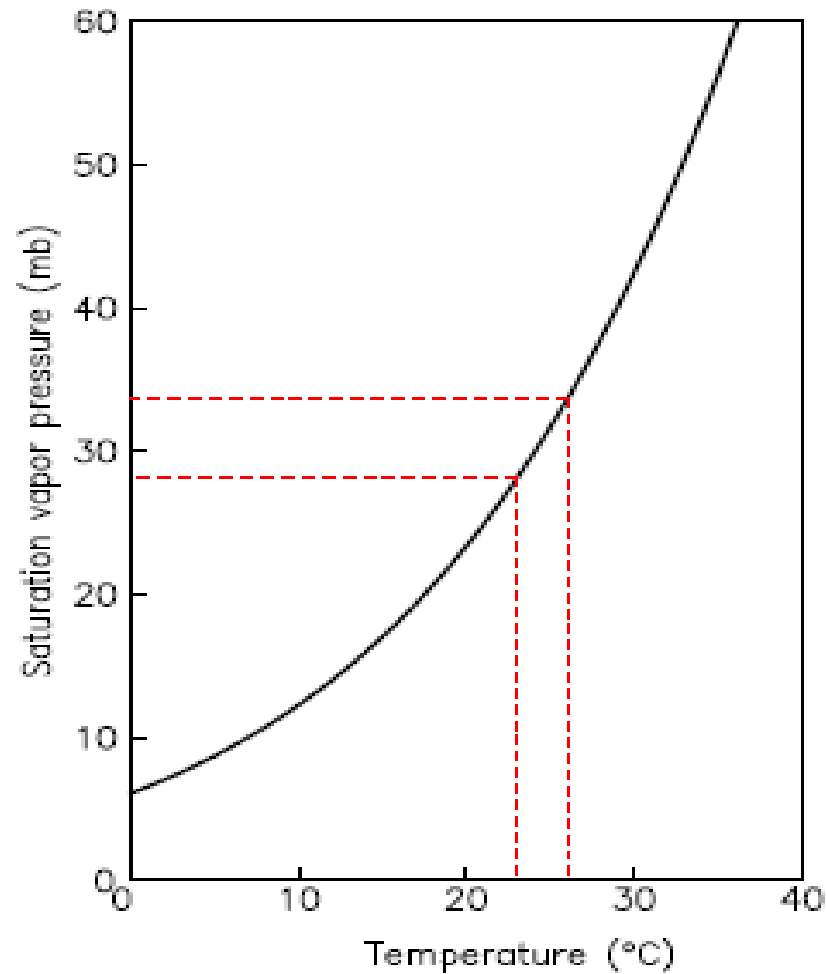
$$\frac{d e_s}{d T} = \frac{L}{T(\alpha_2 - \alpha_1)} ,$$

where L is latent heat constant (for condensation) and α_1 and α_2 are the specific volume of water in gaseous and liquid phases. Within the temperature range of our interest, $e_s(T)$ is close to an exponential function of T if T is in $^{\circ}\text{C}$. For example, the M&P textbook uses the approximation,

$$e_s(T) \approx A \exp(\beta T), \quad \text{where } A = 6.11 \text{ mb}, \beta = 0.067, \text{ and } T \text{ in } ^{\circ}\text{C}$$

Clausius-Clapeyron equation

The saturation vapor pressure increases sharply with temperature;
A small increase in T leads to a large increase in $e_s(T)$



Example:

$$T = 23 \text{ }^\circ\text{C} (296.16 \text{ }^\circ\text{K}) \Rightarrow e_s(T) = 28.1 \text{ mb}$$

$$T = 26 \text{ }^\circ\text{C} (299.16 \text{ }^\circ\text{K}) \Rightarrow e_s(T) = 33.6 \text{ mb}$$

3 °C increase in temperature leads to ~ 20% increase in the saturation vapor pressure. If the air is saturated (e.g., near the surface of the ocean), this implies a 20% increase in the amount of water vapor (in terms of pressure) in the air

$$\text{Within this range, } \frac{\Delta T}{T} \approx \frac{3^\circ K}{296.16^\circ K} \approx 0.01 \text{ , and } \frac{\Delta e_s}{e_s} \approx \frac{5.5 \text{ mb}}{28.1 \text{ mb}} \approx 0.19 \text{ ,}$$

$$\text{therefore } \frac{de_s}{e_s} \gg \frac{dT}{T} \text{ .}$$

More on Clausius-Clapeyron equation

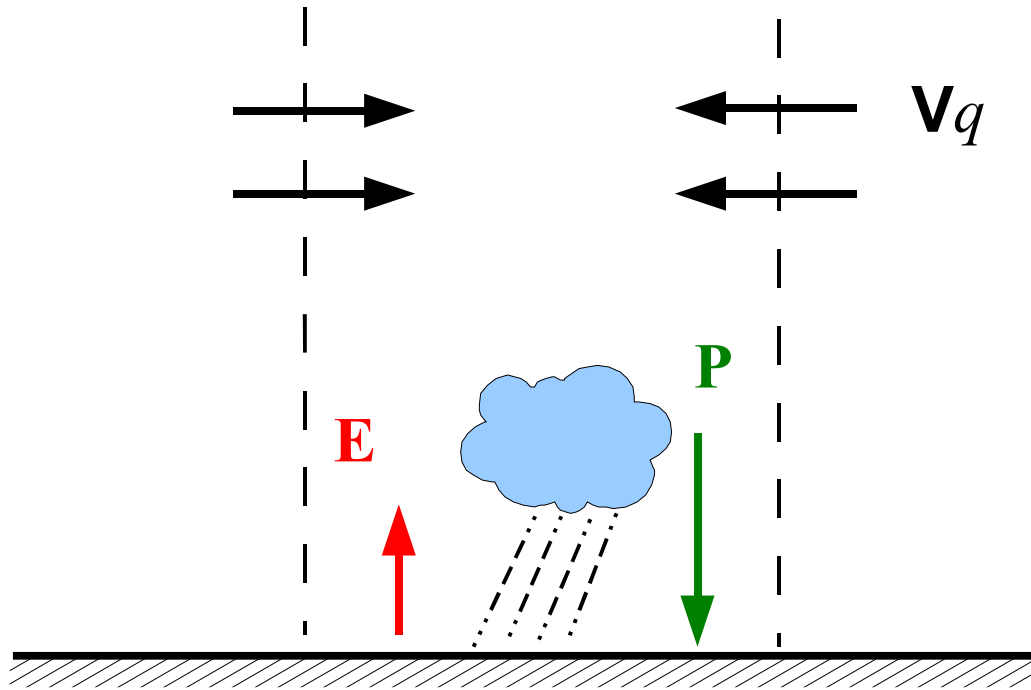
One can think of $e_s(T)$ as the upper bound of e for a given T .

Since $e_s(T)$ decreases sharply with a decreasing temperature, and since atmospheric temperature generally decreases with height within the troposphere, we expect the moisture content in the atmosphere to decrease sharply with height

⇒ **Most of the water vapor is concentrated in the lower troposphere (between the surface and ~ 850 mb)**

Preview of things to come ...

Water balance for an air column

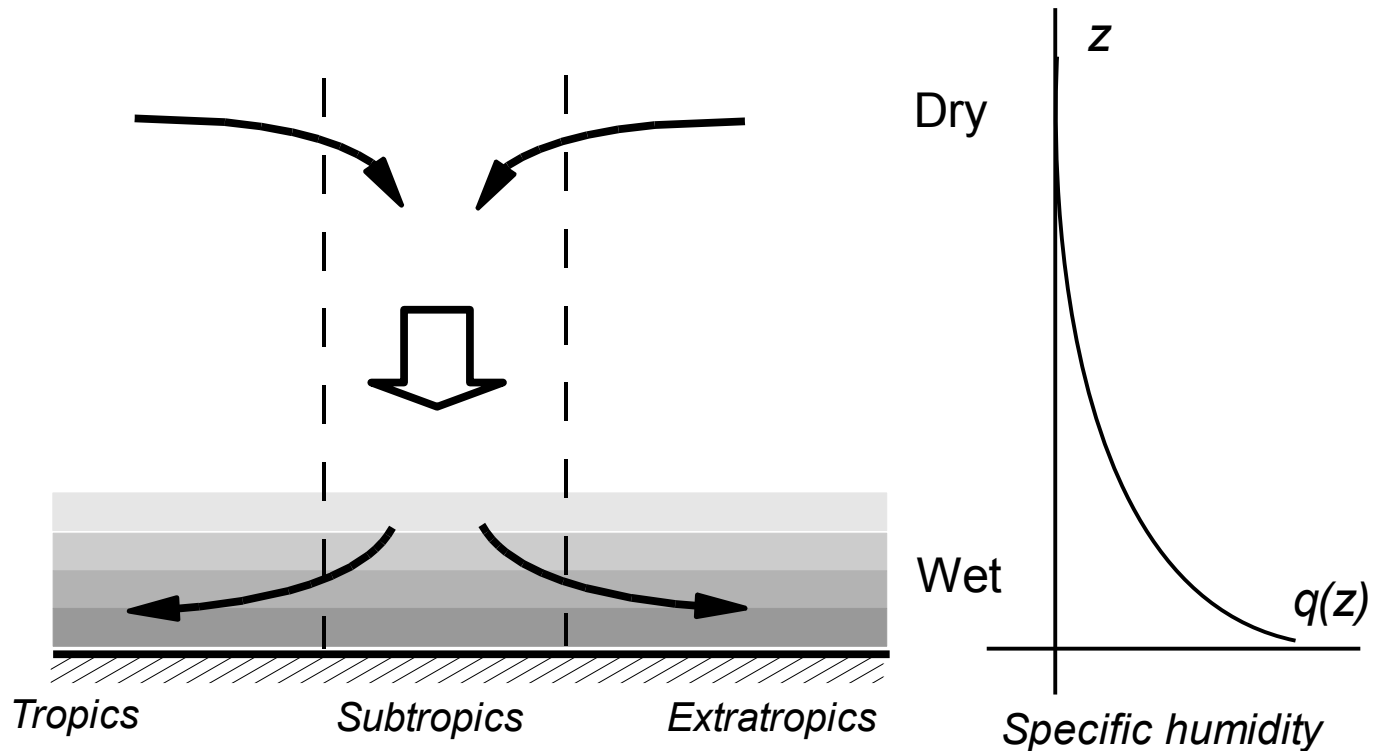


Precipitation – Evaporation = Convergence of moisture

$$P - E = -\int \nabla \cdot (\mathbf{v}q) dz \quad , \quad q \text{ is specific humidity}$$

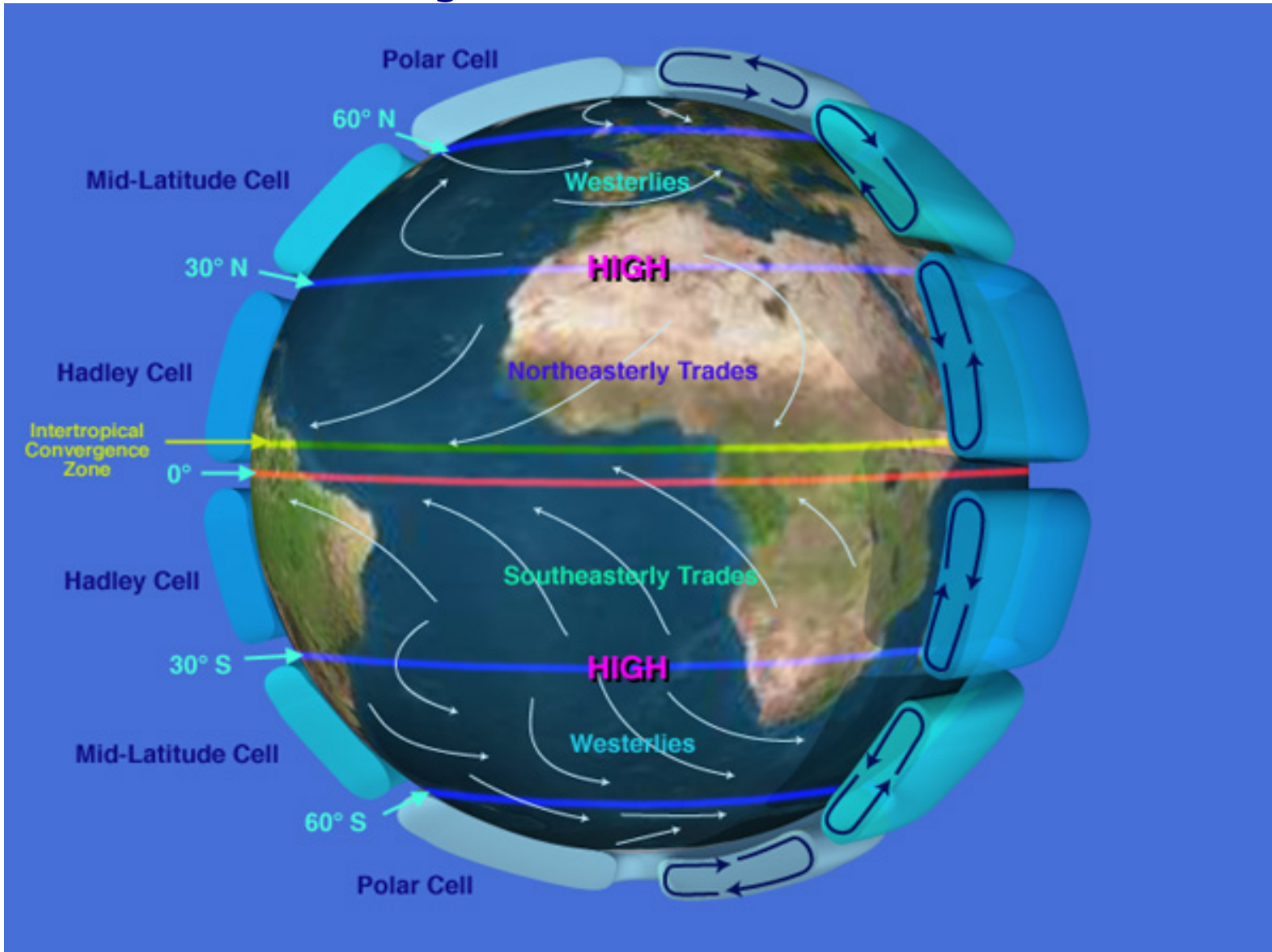
Clausius-Clapeyron related...

Desert climate (subtropics)
Mean subsidence \Rightarrow dry climate



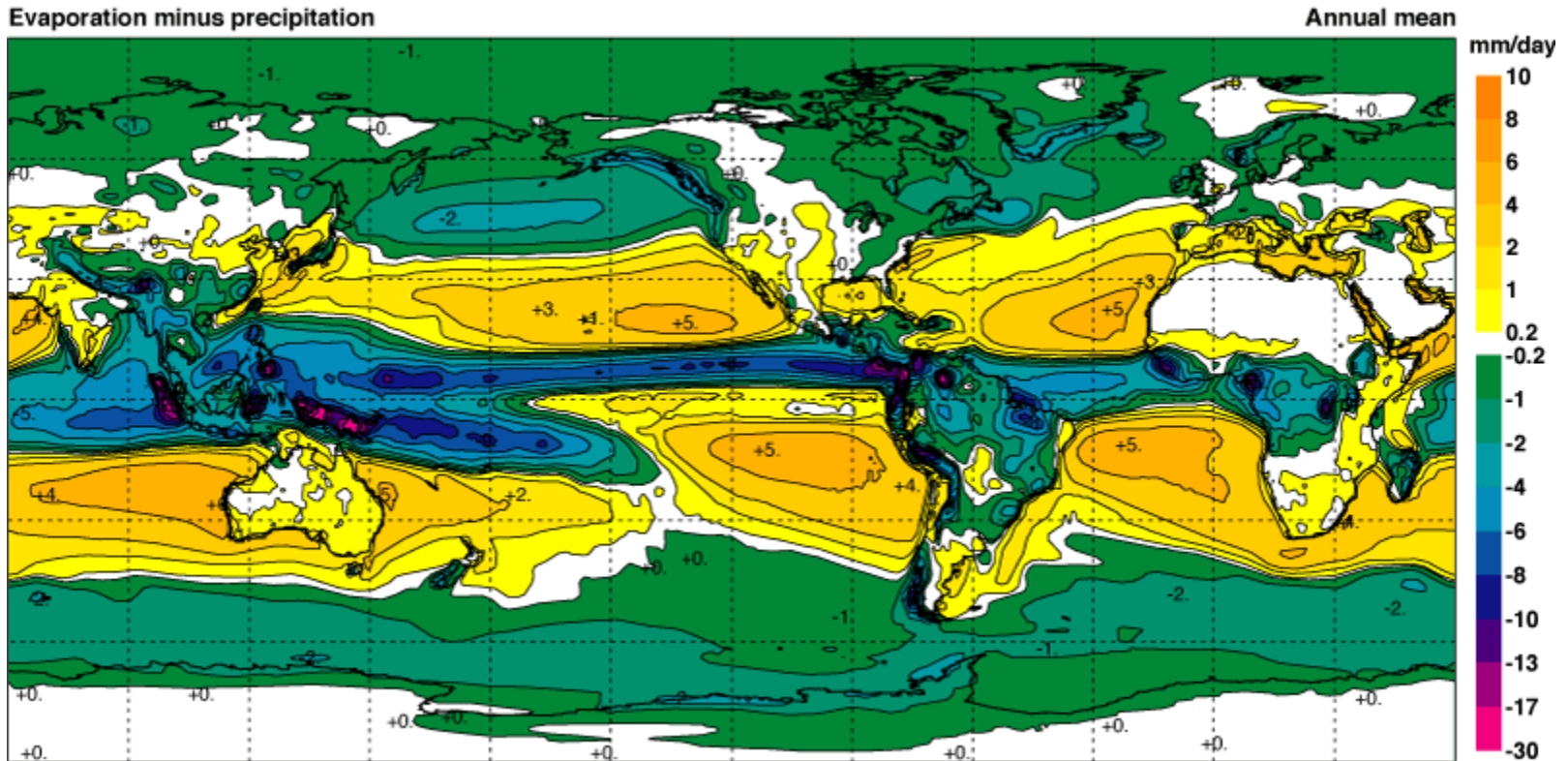
Horizontal mass flux: $in = out$
Horizontal moisture flux: $in < out$
 \Rightarrow *Evaporation exceeds precipitation*

World's major deserts are located in regions with large-scale subsidence



Picture source: NASA JPL

Observed climatology of evaporation minus precipitation (ECMWF Reanalysis, ERA-40 annual mean)



Yellow: Dry zone, $E > P$

Saturation specific humidity

q_s is the critical value such that condensation may occur when $q > q_s$

From Eq. (3) or (4),

$$q_s = \frac{1}{1 + \frac{1}{\varepsilon} \left(\frac{p}{e_s} - 1 \right)} \approx \frac{\varepsilon e_s}{p} . \quad \text{Eq. (5)}$$

Since e_s depends on T , q_s depends on both T and p .

It can also be shown that, under a wide range of conditions,

$$\frac{dq_s}{q_s} \gg \frac{dT}{T} . \quad \text{Eq. (6)}$$

(It is left to you as an exercise to show that Eq. (6) is a good approximation. This is also related to Prob 6 of Ch.4 in M&P textbook.)

Relative humidity

$$RH \equiv q / q_s$$

Condensation can begin to happen when $RH \rightarrow 1$

Sometimes (especially when the air is very clean - no "cloud condensation nuclei"), one can observe the situation that condensation does not occur until $RH > 1$ (supersaturation). We will skip this detail for now.