MAE 598 Fall 2012 Homework #3
This is an open draft. More detail and hints will be added if necessary.

The difficulty level of this exercise is "**D**". You will work on it for bonus. You may still receive an "A" grade for the course without turning in a satisfactory answer for this homework, provided that you do well on most assignments with difficulty level "**E**" and "**M**" and submit a high quality final report.

<u>You have an option of expanding this exercise into your final project</u>.

1. The hydrostatic version of the governing equation of an atmospheric flow in p-coordinate is given as

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\partial \Phi}{\partial x} + f v + F_{r,x} \qquad (1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\partial \Phi}{\partial y} - f u + F_{r,x} \qquad (2)$$

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p} \qquad (3)$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + (\gamma - \gamma_d)\omega + \dot{Q}/c_p \qquad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad (5)$$

[*An equation for surface pressure,* p_S] , (6)

where Φ is geopotential (≡ gz where z is height), T is temperature, $\omega \equiv dp/dt$ is the "vertical velocity" in p-coordinate, and (u, v) is the horizontal velocity on a constant-pressure surface. In addition, $\gamma \equiv \partial T / \partial p$ is the vertical lapse rate of the environmental temperature and $\gamma_d \equiv RT / p c_p$ is the dry adiabatic lapse rate, both in p-coordinate; $f = 2\Omega \sin(\phi)$ is the Coriolis parameter where Ω is the rotation rate of the Earth and ϕ is latitude; ($F_{r,x}$ $F_{r,y}$) is friction for horizontal momentum; $\dot{Q}/c_p$ is diabatic heating/cooling. Your task is to use these equations as the basis to make a 6-hour forecast of the (vertical profiles of) temperature, geopotential, and horizontal velocity at a grid point in the middle of the Pacific Ocean and validate them with observation. Specifically, you will use 00Z, January 1, 2012 as the starting time to make the forecast for 06Z, January 1, 2012. The observations to be used for the initial condition and for validation will come from the 4-times daily NCEP Reanalysis II. They are freely available from NOAA ESRL/PSD website (http://www.esrl.noaa.gov/psd) but we will assist you in obtaining the data if needed.

Remarks:

(1) It is understood that Eq. (3) will be used to obtain Φ by a vertical integration of temperature,

$$\Phi(p) = \Phi_S + \int_{p_s}^{p} RT\, d\ln p \quad . \qquad (3A)$$

To do so, we need the surface height (topography), Φ_S (which can be set to zero over the ocean), and surface pressure. The surface pressure at the initial time is available from the Reanalysis. In principle, we need to update as the forecast progresses. See Remark #3 for suggestions.

(2) It is understood that Eq. (5) will be used to obtain w by a vertical integration of the horizontal divergence, either from the top or from the bottom,

$$\omega(p) = \int_0^p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp \quad , \qquad (5A)$$

or

$$\omega(p) = \omega_S + \int_{p_S}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp \quad . \qquad (5B)$$

Equation (5A) is slightly simpler (as it does not involve surface pressure). For this exercise, you may adopt it instead of (5B).

(3) After the initial time, we face the problem of updating the surface pressure which is needed in Eq. (3A) (or (5B) if you prefer to use it). For this exercise, this can be overcome at several different levels, from the simplest (but least realistic) to the most sophisticated:

(i) Keep p_S at its initial value through the whole duration. This is trivial to do but is ultimately unrealistic. During the passage of a winter storm (or during the developing stage of the storm) surface pressure can drop substantially over a 6-hour period. Nevertheless, if you are unable to execute the forecast using the more sophisticated methods, this is better than nothing.

(ii) Take the observed p_S at the initial and final times (00Z and 06Z of January 1, 2012), linearly interpolate it onto every time step and use it to advance the numerical integration. This is cheating since we are not supposed to know the at 06Z, January 1, 2012! (If we already know about it, we do not need the forecast.) Nevertheless, for the purpose of getting through this exercise, you are allowed to adopt this approach if all else fail. Like approach (i), at least it is better than not doing the "forecast" at all.

(iii) At each time step with t > 0, use the multi-level Φ and T obtained from the numerical integration to calculate p_S by interpolation or extrapolation (using the hydrostatic equation). We will provide supplementary information for this approach.

(iv) Directly predict p_S at every time step. In this case, we need an equation for the tendency of p_S. This may be constructed by first considering the continuity equation but integrating it from the surface to the top of the atmosphere:

$$\omega(p_S) = \int_0^{p_S} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp \quad . \qquad (7)$$

Next, note that (by definition)

$$\omega(p_S) \equiv \frac{d p_S}{dt} \equiv \frac{\partial p_S}{\partial t} + \boldsymbol{V}_S \cdot \nabla_H p_S \quad , \qquad (8)$$

where $\boldsymbol{V}_S$ is the horizontal velocity at surface and $\nabla_H p_S$ is the horizontal gradient of surface pressure (note that the term for the vertical advection, $w_S \partial p_S / \partial z$, is zero since $w_S = 0$.) If we reasonably assume $\boldsymbol{V}_S \cdot \nabla_H p_S = 0$, then the combination of (7) and (8) leads to

$$\frac{\partial p_S}{\partial t} = \int_0^{p_S} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp \quad . \qquad (6A)$$

At any time t, one can use the information of p_S (t), u(t), and v(t) to evaluate the right hand side, then integrate the equation in time to obtain p_S (t+Δt).

(4) In Eq. (1-6) we have used the Cartesian coordinate (x,y) for conciseness. In practice, beware that spherical geometry has to be taken into account. The most rigorous way to do so is to rewrite the equations in spherical coordinate (see Chapter 2 of textbook). For the forecast made for a single grid point, however, this is not a big concern. One can keep the x and y coordinate but carefully take into account the fact that Δx shrinks with latitude.

(5) You might start with a large Δt (say 1 hour). If the numerical integration blows up, try a smaller value (say Δt = 30 minutes). Note that with a smaller Δt you will have to integrate the system for more time steps to complete the 6-hour forecast. This implies a bigger "domain of dependence". namely, you will need to use more grid points in the neighborhood of the target grid point to perform the numerical integration.

(6) As a first attempt at solving the problem, please ignore all frictional and diabatic terms. Just set ($F_{r,x}$ $F_{r,y}$) and $\dot{Q}/c_p$ to zero. Later, if you decide to expand the exercise into a term project, we can explore the possibility of imposing a parameterized form of of those terms in the governing equation.

(7) Feel free to use any finite-difference schemes to discretize the terms in the right hand side of Eqs. (1)-(6). Through this exercise, you will relive what Richardson (1922) might have gone through in his pioneering attempt at weather forecast, except that (with a great hindsight) you are provided with a set of equations that are somewhat "safer" to integrate in time.

(8) The Reanalysis data are in netCDF format. Supplementary material will be provided for the Matlab functions that can be used to read the data. (We also have its equivalent for Fortran or C on a Linux platform if you prefer to use either.)