

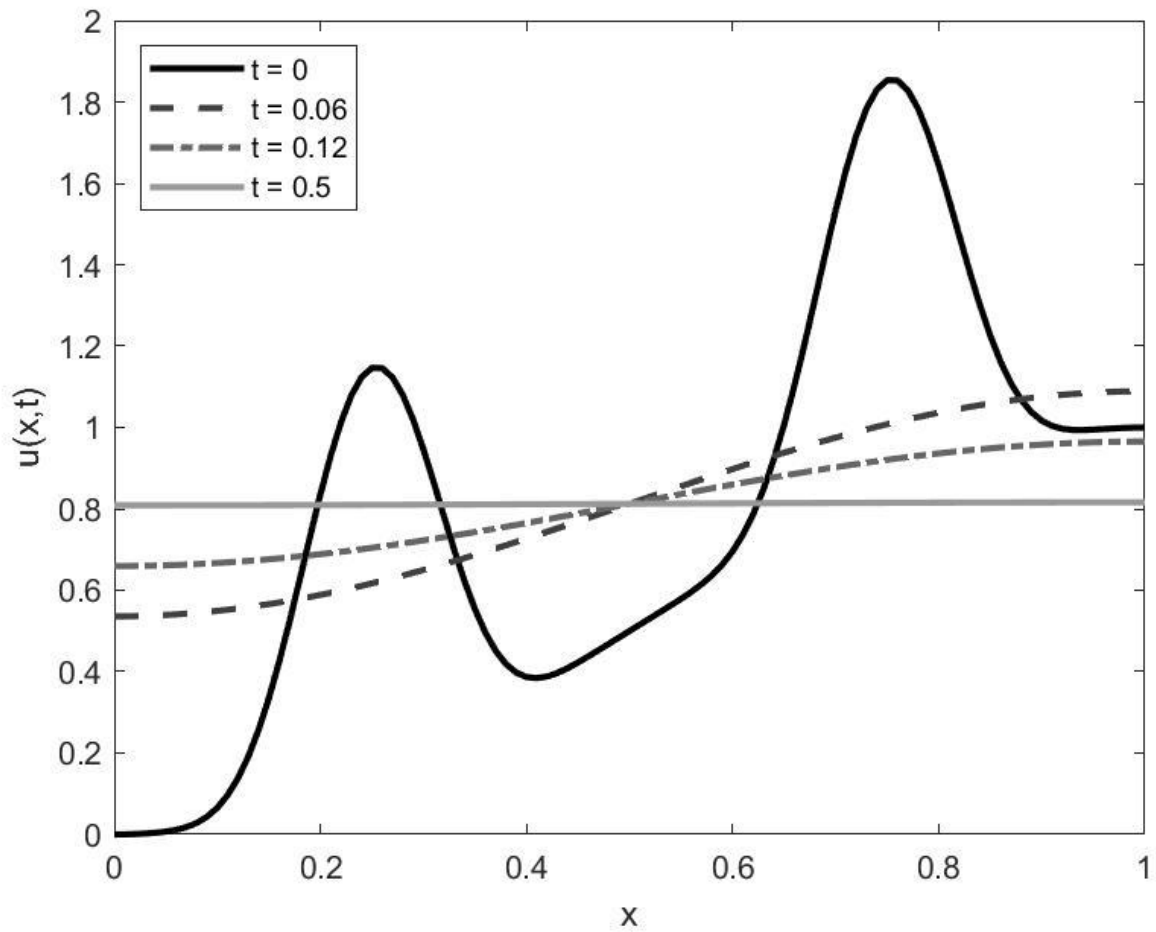
MAE/MSE 502 Fall 2019 HW1 Solution

Prob 1(a)

$$u(x, t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$$a_0 = \int_0^1 P(x) dx, \text{ and } a_n = 2 \int_0^1 P(x) \cos(n\pi x) dx \text{ for } n > 0,$$

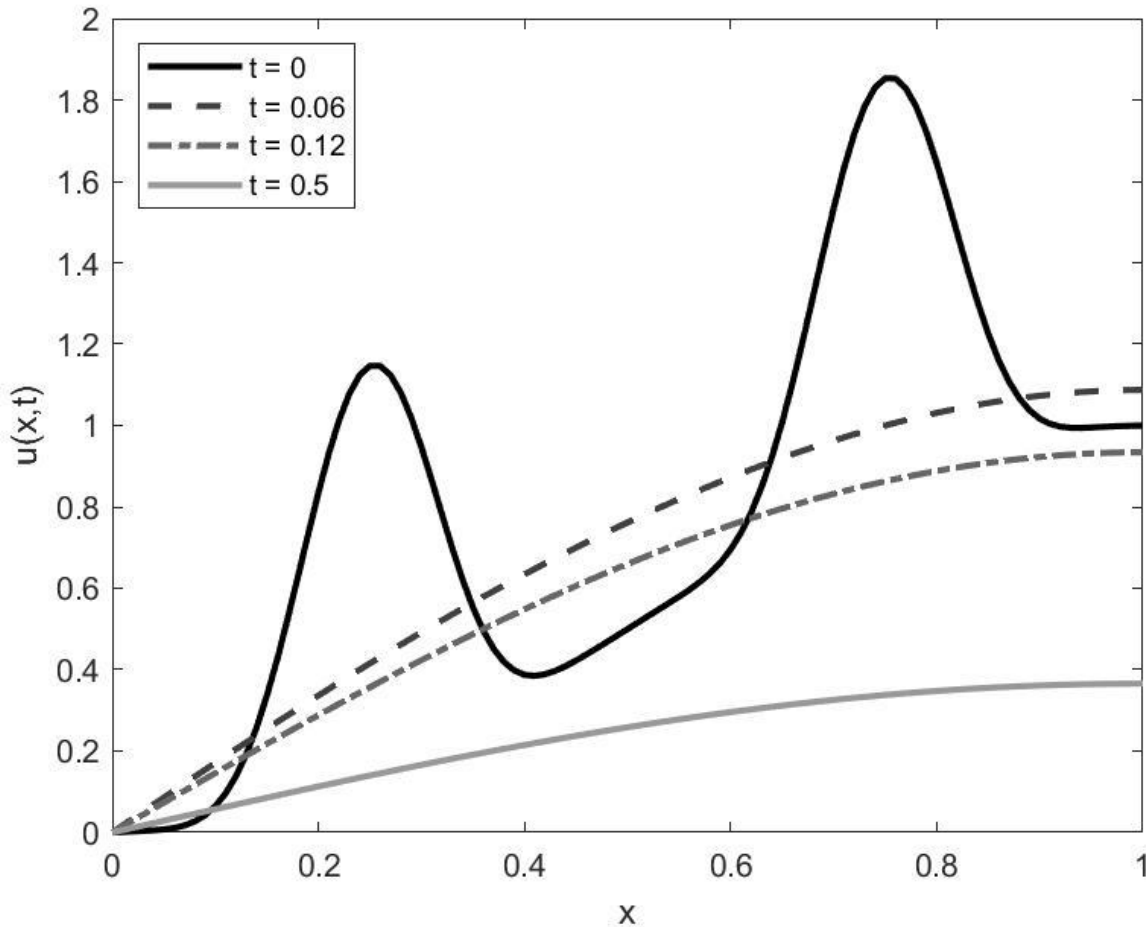


Prob 1(b)

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t\right)$$

where the summation is over odd values of n only, and

$$a_n = 2 \int_0^1 P(x) \sin\left(\frac{n\pi x}{2}\right) dx, \text{ for } \underline{\text{odd values of } n}.$$



An example of Matlab code for making the plot for Prob 1(b) is given in the next page. In the code, the function $\text{mod}(M, N)$ is the remainder of M divided by N . For example, $\text{mod}(12, 5) = 2$ since the remainder of 12 divided by 5 is 2. With that, $\text{mod}(n, 2) = 0$ if n is even, and $\text{mod}(n, 2) = 1$ if n is odd. This is used to eliminate all a_n with even values of n .

The code also illustrates how to make a clear black-and-white plot. With multiple lines, they can be distinguished by the line pattern and gray scale.

```

clear
x = [0:0.005:1]; t = [0 0.06 0.12 0.5]; N = 30;
p = (sin(2*pi*x)).^6+(sin((pi*x)/2)).^2;
for n = 1:N
    a(n) = mod(n,2)*2*trapz(x,sin(n*pi*x/2).*p);
end
for m = 1:length(t)
    u(m,:) = 0*x;
    for n = 1:N
        u(m,:) = u(m,.)+a(n)*sin(n*pi*x/2)*exp(-t(m)*(n*pi/2)^2);
    end
end
end
hold on
plot(x,u(1,:),'-','Color',[0 0 0],'LineWidth',2)
plot(x,u(2,:),'--','Color',[0.25 0.25 0.25],'LineWidth',2)
plot(x,u(3,:),'-.','Color',[0.4 0.4 0.4],'LineWidth',2)
plot(x,u(4,:),'-','Color',[0.6 0.6 0.6],'LineWidth',2)
xlabel('x'); ylabel('u(x,t)');
legend('t = 0','t = 0.06','t = 0.12','t = 0.5','Location','NorthWest')
box on
hold off

```

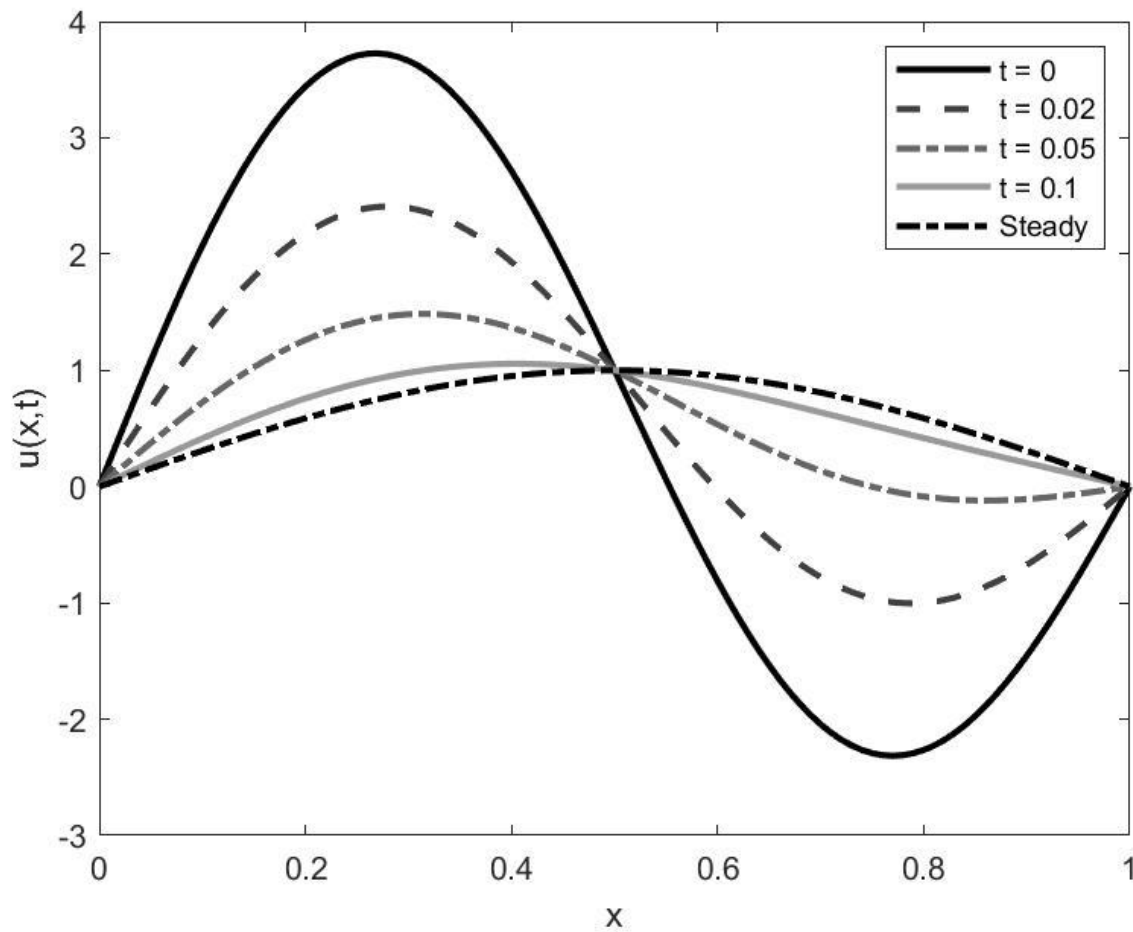
Prob 2

$$u(x, t) = \sin(\pi x) + 3 \sin(2\pi x) e^{-3\pi^2(1-e^{-t})}$$

Steady solution is

$$u_s(x) = \sin(\pi x) + 3 \sin(2\pi x) e^{-3\pi^2}$$

Plot:



Prob 3

$$u(x, t) = e^{-t^3/3} + \cos(\pi x) e^{-(\pi^2 t^2/2 + t^3/3)}$$

Prob 4

We will discuss the solution of this problem in class.