MAE/MSE 502, Fall 2019 HW2 Solution

Prob 1

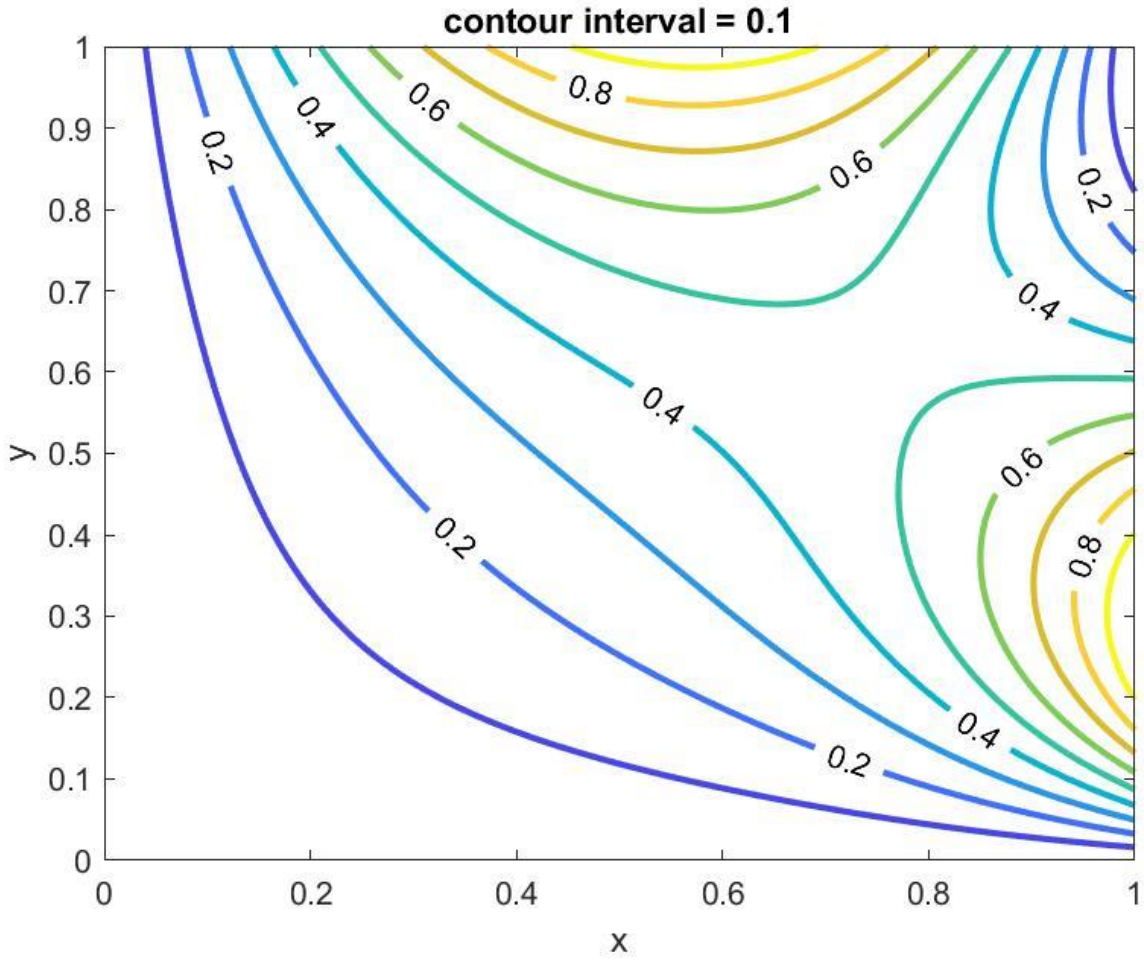
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y)\sinh(n\pi x) + b_n \sin(n\pi x)\sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin((1-y)^2\pi)\sin(n\pi y)\, dy$$

$$b_n = \frac{2}{\sinh(n\pi)} \int_0^1 2.5\,(x - x^3)\sin(n\pi x) dx$$

Plot:



Prob 2

(a) There are infinitely many solutions

(b)

$$u(x,y) = C + 3x + \frac{1}{2\pi}[\sinh(2\pi x) - \coth(2\pi)\cosh(2\pi x)]\cos(2\pi y)$$

where *C* is an arbitrary constant (which corroborates the conclusion from Part (a)).

Prob 3

$$u(x,y) = \left(2x^4 - \frac{2}{x}\right)\sin(2y)$$

Prob 4

$$u(x,y) = e^{3x} + \left(\frac{e^{4x} - e^{-x}}{e^4 - e^{-1}}\right)\cos(2y)$$