MAE/MSE 502, Fall 2019 HW3 Solution

Prob 1

(a) Not a Sturm-Liouville system

(b) There is only one eigenvalue, $c = [ln(2+\sqrt{3})]^2 \approx 1.7344$. The associated eigenfunction is $G(x) = A\sinh(\sqrt{c}\,x)$, where A is an arbitrary constant. (The eigenvalue can also be written as $c = [\cosh^{-1}(2)]^2$.)

Prob 2

(a)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\cos(n\pi t)$$

where

$$a_n = 2\int_0^1 \left(\sqrt{x} - x\right)^2 \sin(n\pi x)\,dx$$

(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\sin(n\pi t)$$

where

$$a_n = \frac{2}{n\pi}\int_0^1 \left(\sqrt{x} - x\right)^2 \sin(n\pi x)\,dx$$

Prob 3

$$u(x,t) = \sum_{n=-\infty}^{\infty} C_n(0)e^{(inU - n^2K)t + inx}$$

where

$$C_n(0) = \frac{1}{2\pi}\int_0^{2\pi} u(x,0)e^{-inx}dx$$

Prob 4

$$u(x,t) = \frac{1}{3}\sinh(3t)\cos(x) + \sin(2x)$$

Prob 5

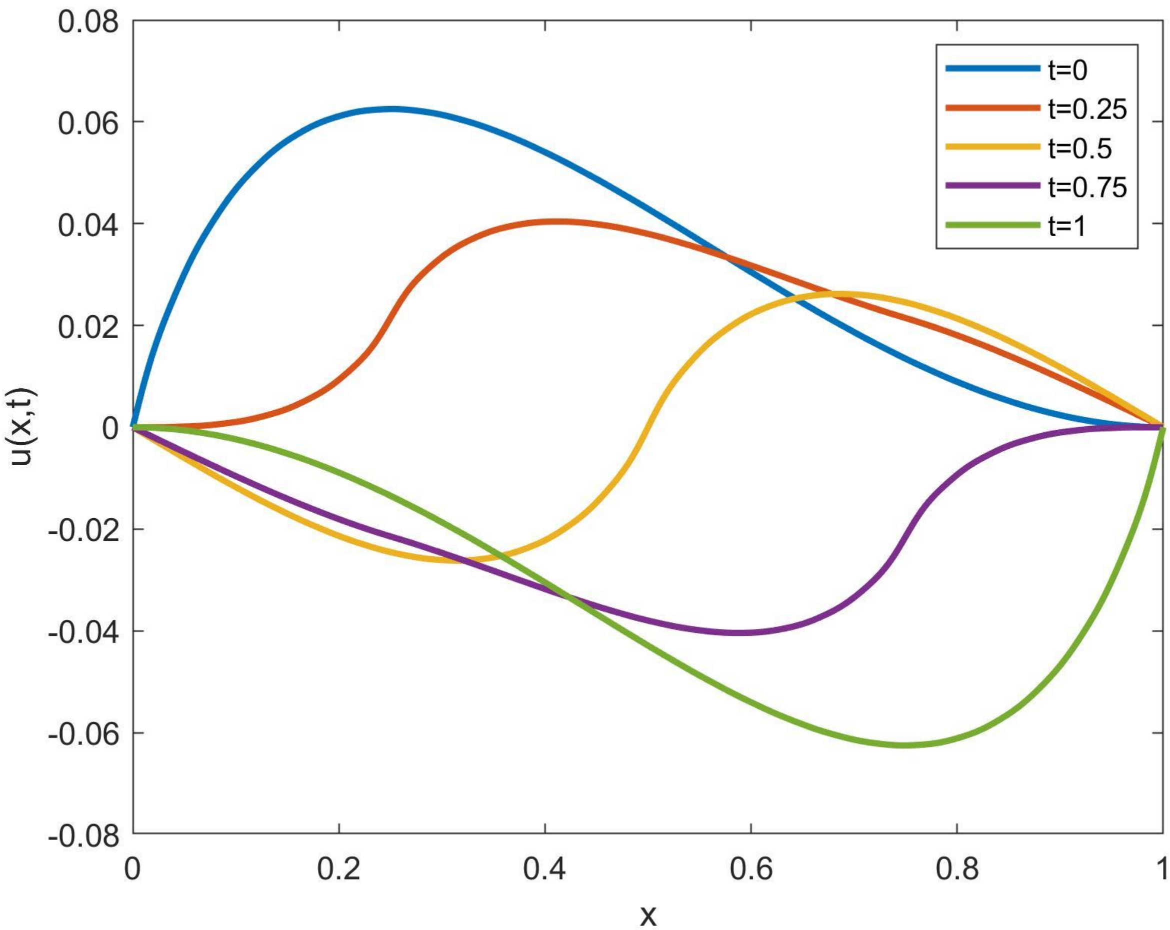
$$u(x,t) = e^{t^2} + \sin\left(x - \frac{t^2}{2}\right)$$

Prob 6

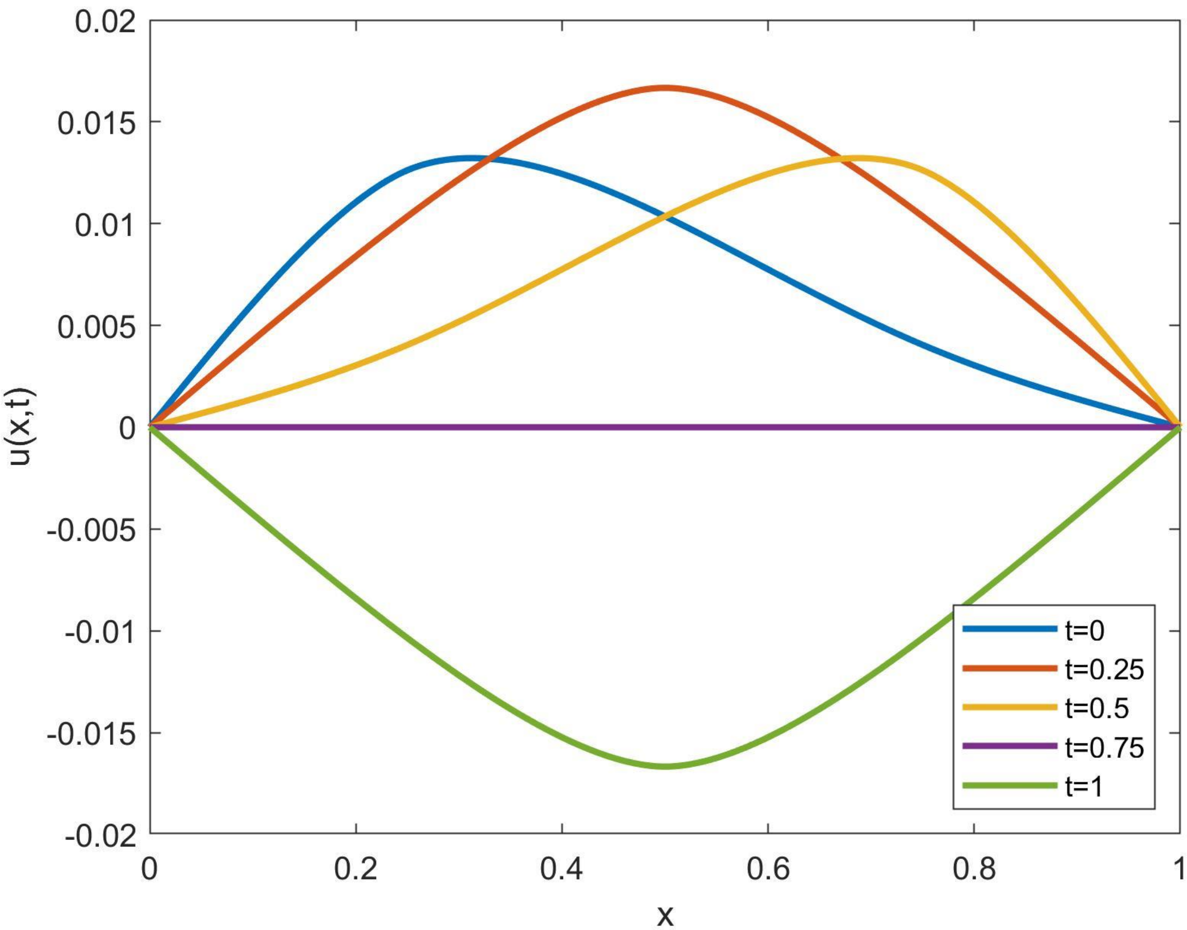
$$u(x,t) = \cosh(t) + \sin(x+t) - \sin(x)$$

See next 3 pages for plots

Plot for Prob 2a



Plot for Prob 2b



Plot for Prob 3