MAE/MSE 502, Fall 2019 Homework #5

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration. **For ALL problems in this homework, we expect a closed-form solution without any unevaluated integrals.**

Prob 1 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = t\,u$$

with the boundary condition

$$u(x,0) = e^{-x^2} .$$

Prob 2 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} + u = 2t - x$$

with the boundary condition,

$$u(x,0) = x .$$

Prob 3 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} + (2+2x)\frac{\partial u}{\partial x} = (3+3u)$$

with the boundary condition

$$u(x,0) = e^{-x^2}$$

Prob 4 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + t\frac{\partial u}{\partial x} = 3$$

with the boundary condition,

$$u(x,0) = \begin{cases} -\frac{1}{x}\,, if\ x \leq -1 \\ \\ 1\ \,, if\ x > -1 \end{cases}$$

Plot the solution as a function of x at $t = 0.1$ and 0.3, and the initial state, $u(x, 0)$. Please make the plot over the range of $-5 \leq x \leq 5$ and collect all 3 curves in one plot.