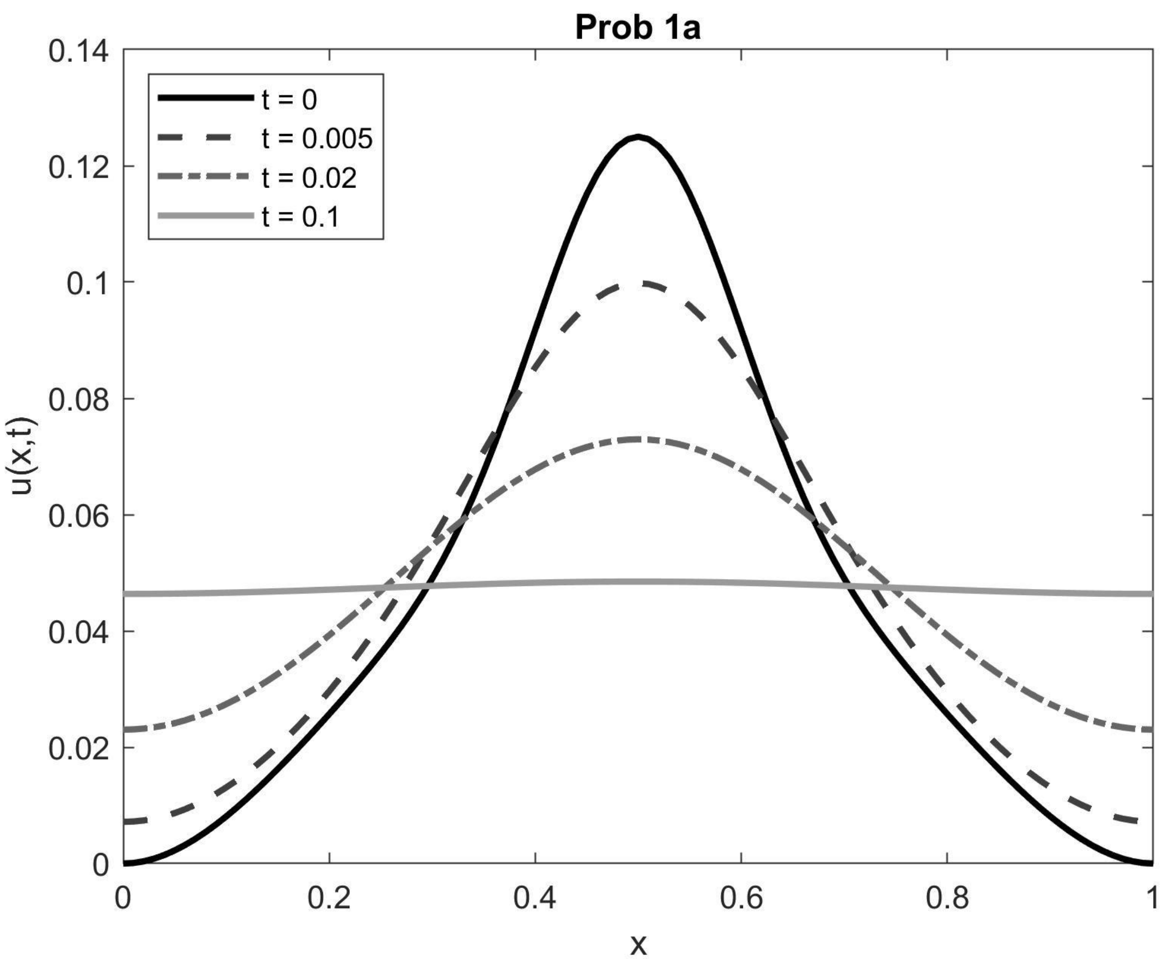
MAE/MSE 502 Fall 2020 HW1 Solution

Prob 1(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)\exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\,dx$, and $a_n = 2\int_0^1 P(x)\cos(n\pi x)dx$ for $n > 0$,

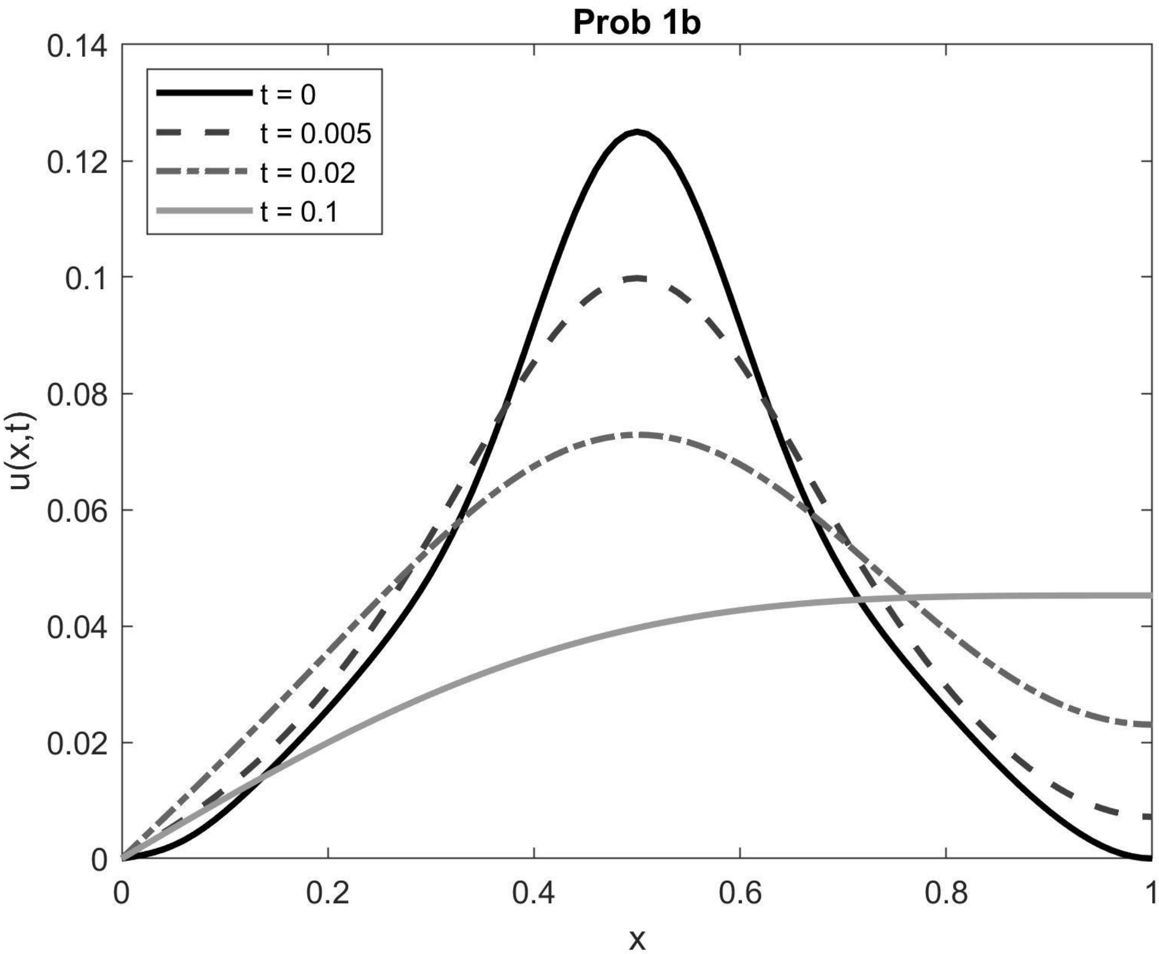


Prob 1(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi x}{2}) \exp(-(\frac{n\pi}{2})^2 t)$$

where the summation is over <u>odd values of *n* only</u>, and

$a_n = 2\int_0^1 P(x)\sin(\frac{n\pi x}{2})dx$, for <u>odd values of *n*</u>.

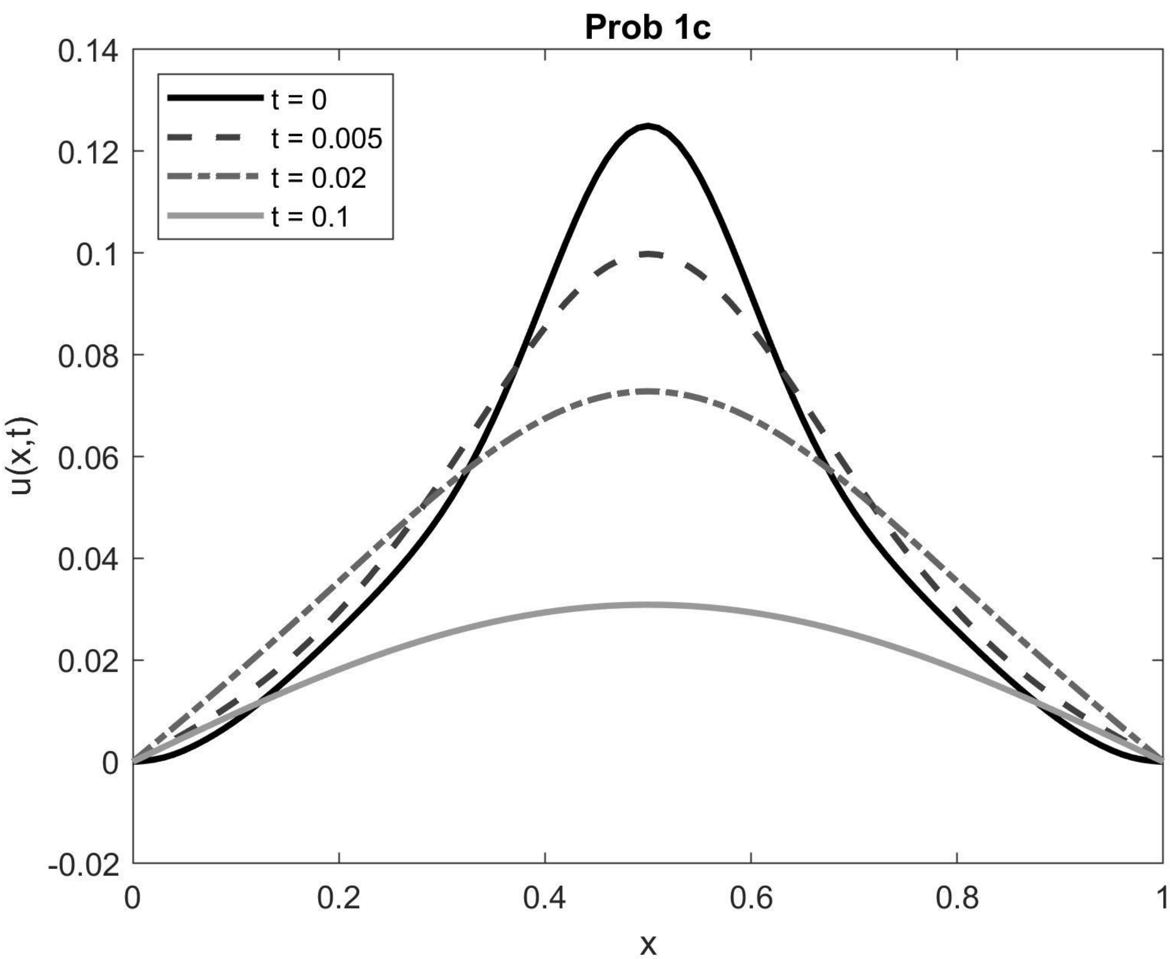


Prob 1(c)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\,dx$, and $a_n = 2\int_0^1 P(x)\sin(n\pi x)dx$ for $n > 0$,



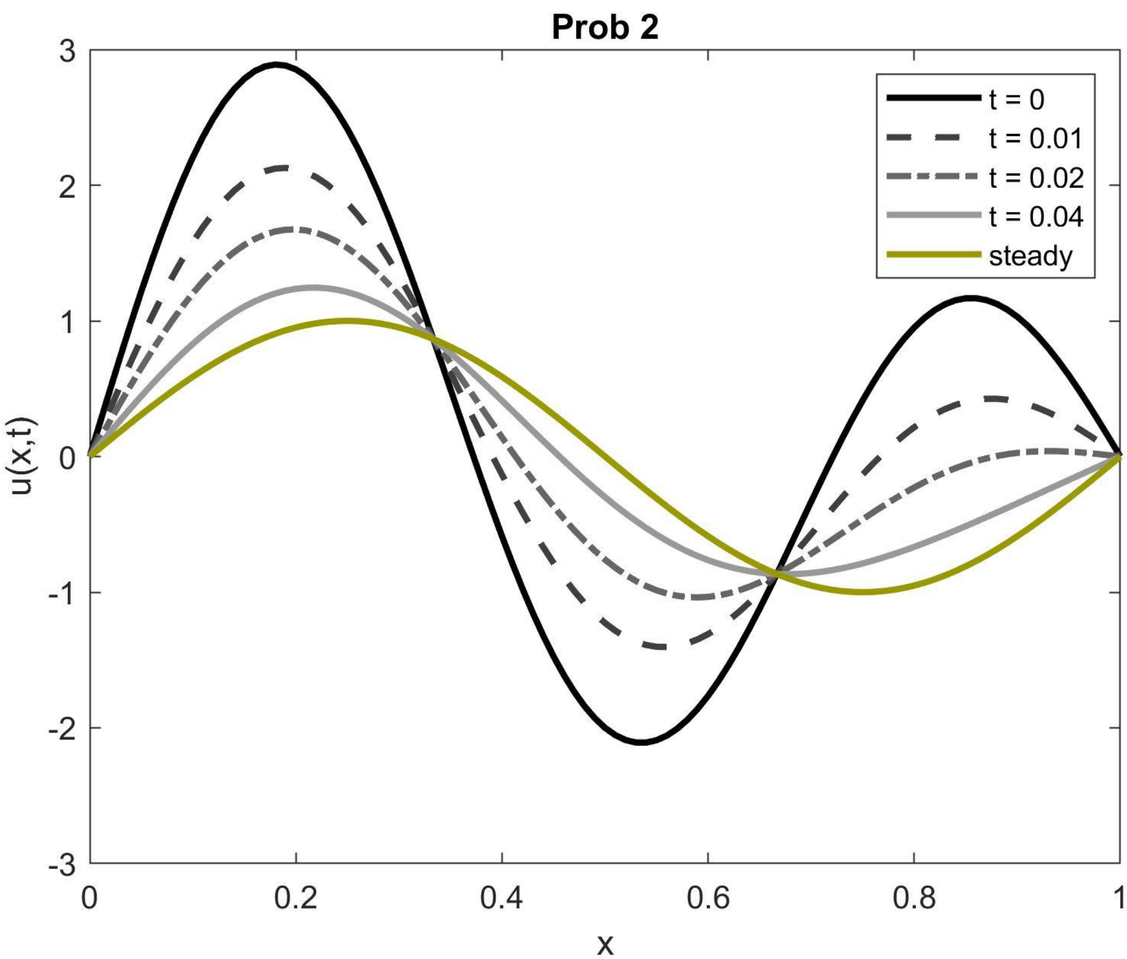
Prob 2

$$u(x,t) = \sin(2\pi x) + 2\sin(3\pi x)\, e^{-5\pi^2\left(1-\frac{1}{1+t}\right)}$$

Steady solution is

$$u_s(x) = \sin(2\pi x) + 2\sin(3\pi x)\, e^{-5\pi^2}$$

Plot:



Prob 3

$$u(x,t) = e^{1-\cos(t)} + \cos(2\pi x)\, e^{1-\cos(t)-4\pi^2\left(t+\frac{t^2}{2}\right)}$$

Prob 4

We will discuss the solution of this problem in class.