MAE/MSE 502, Fall 2020 HW2 Solution

Prob 1

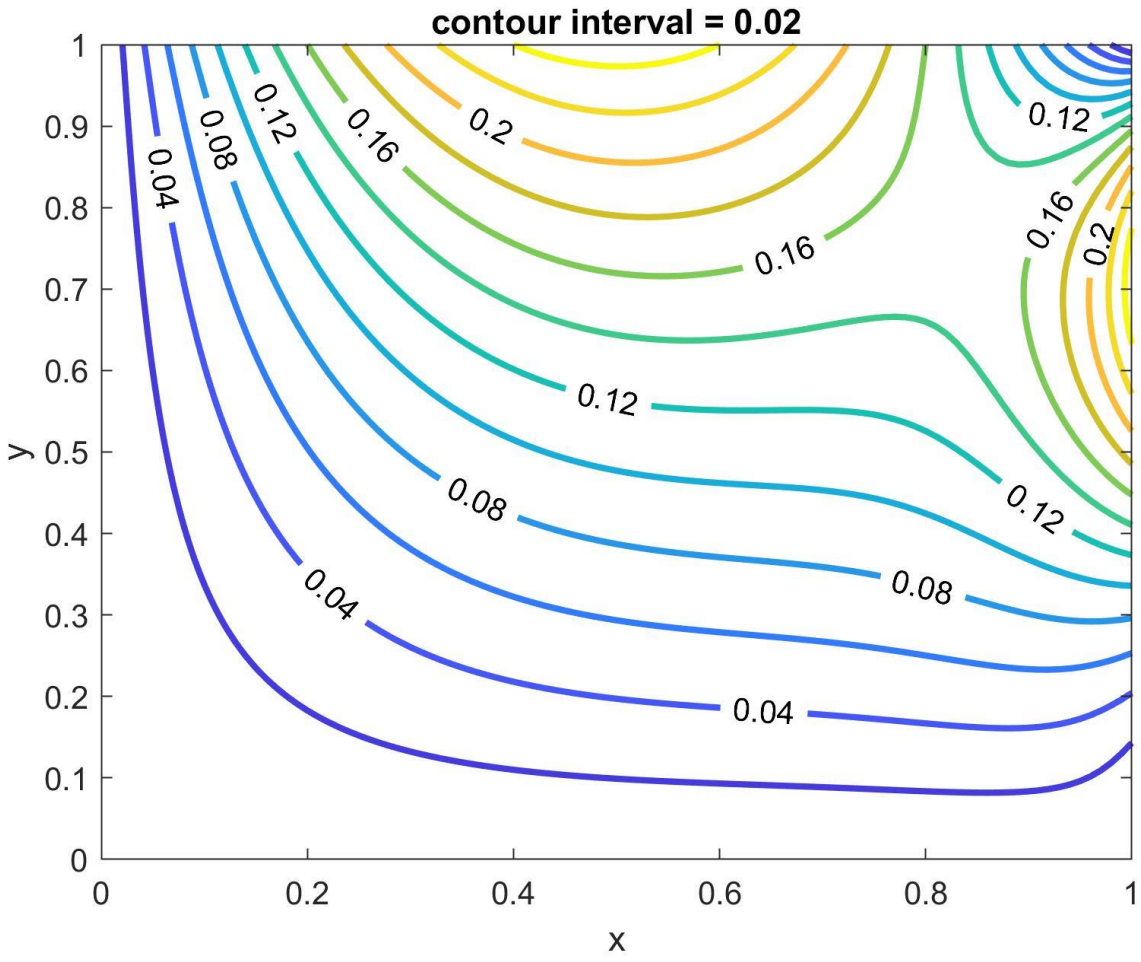
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\sinh\left(\frac{n\pi y}{2}\right) + b_n \sin(n\pi y)\sinh(2n\pi x)$$

where

$$a_n = \frac{2}{\sinh\left(\frac{n\pi}{2}\right)}\int_0^1 (x - x^2)\sin(n\pi x)dx$$

$$b_n = \frac{2}{\sinh(2n\pi)}\int_0^1 (y^2 - y^4)\sin(n\pi y)\,dy$$

Plot:



Prob 2

(a) Solvability condition is satisfied. There are infinitely many solutions.

(b)

$$u(x,y) = C + 5x + \frac{1}{3\pi}\left[\sinh(3\pi x) - \frac{\cosh(3\pi)}{\sinh(3\pi)}\cosh(3\pi x)\right]\cos(3\pi y)$$

where *C* is an arbitrary constant (which corroborates the conclusion from Part (a)).

Prob 3

$$u(x,y) = \left(\frac{4}{y^2} + y^4\right)\sin(3x)$$

Prob 4

$$u(x,t) = \cosh(t) + t\cos(x) + \frac{1}{\sqrt{3}}\sin(\sqrt{3}\,t)\cos(2x)$$

The detail of the solution for Prob 2, 3, and 4 will be posted separately.