MAE/MSE 502, Fall 2020, Homework # 3

The report for this assignment is due (Arizona time) 11:59 PM, Tuesday, November 17th, at Canvas. The report should include a statement on collaboration, and computer code(s) used for the assignment. Please follow the rules for collaboration as given in the first page of the problem statement of Homework #1.

Prob 1 (4 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, consider the PDE (in which U and K are constants)

$$\frac{\partial u}{\partial t} = U\frac{\partial u}{\partial x} + K\frac{\partial^2 u}{\partial x^2}$$

with periodic boundary conditions in the x-direction (i.e., $u(0, t) = u(2\pi, t)$, $u_x(0, t) = u_x(2\pi, t)$, and so on), and the boundary condition in the t-direction given as

$$u(x, 0) = \exp[(1 - \cos(x))^2] \quad .$$

Solve the PDE by Fourier series expansion. Plot the solution $u(x, t)$ at $t = 0.1$ for the three cases with (i) $U = 12$, $K = 0$, (ii) $U = 0$, $K = 2.5$, and (iii) $U = 12$, $K = 2.5$. Also, plot the solution at $t = 0$ (which is the same for all three cases). Please collect all four curves in one plot.

Prob 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = 4t\,\frac{\partial^2 u}{\partial x^2} + t\,\frac{\partial^3 u}{\partial x^3} + (1+t)\frac{\partial^4 u}{\partial x^4} - 16\,u$$

with periodic boundary conditions in the x-direction, and the boundary condition in the t-direction given as

$$u(x, 0) = 1 + \sin(2x) \, .$$

We expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be expressed in real numbers and functions. Expect a deduction if these requirements are not satisfied.

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = 9\,\frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + 8\,u$$

with periodic boundary conditions in the x-direction and the following boundary conditions in the t-direction,

(i) $u(x, 0) = \cos(3x)$,

(ii) $u_t(x, 0) = \sin(x)$.

We expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be expressed in real numbers and functions. Expect a deduction if these requirements are not satisfied.