MAE/MSE 502, Fall 2020 Homework #4

The report for this assignment is due (Arizona time) 11:59 PM, Sunday, November 29th, at Canvas. The report should include a statement on collaboration, and computer code(s) used for the assignment. Please follow the rules for collaboration as given in the first page of the problem statement of Homework #1.

For all problems in this homework, we expect a closed-form exact solution with only a finite number of terms and without any unevaluated integral. The solution, $u(x,t)$, should be expressed explicitly in real functions of x and t and real numbers. Expect a deduction if these requirements are not satisfied.

Prob 1 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t\, e^{-4\pi^2 t} \cos(2\pi x) + \sin(t)$$

with the boundary conditions

(i) $u_x(0,t) = 0$ (ii) $u_x(1,t) = 0$ (iii) $u(x,0) = \cos(2\pi x)$.

Prob 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \cos(x)$$

with the boundary conditions,

(i) $u(0, t) = 0$ (ii) $u(\pi, t) = 2$ (iii) $u(x, 0) = 1 - \cos(x) + \sin(x)$

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + \sin(x)\sin(t) + \cos(2x)$$

with periodic boundary conditions in x-direction, and the boundary conditions in t-direction given as

(i) $u(x,0) = \cos(x)$

Prob 4 (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 9u + 9 + t\sin(3x)$$

with periodic boundary conditions in the x-direction, and the boundary conditions in the t-direction given as

(i) $u(x,0) = 0$ (ii) $u_t(x,0) = 1$