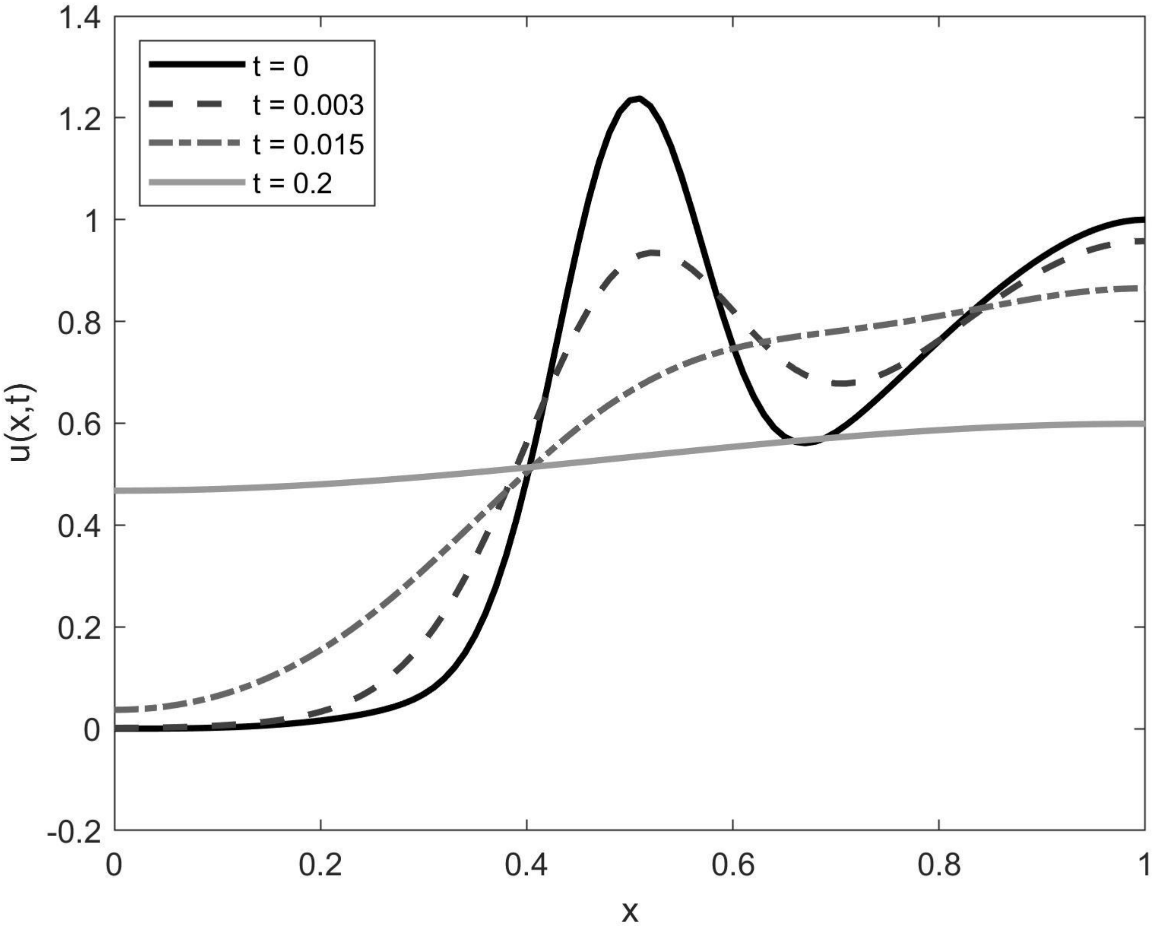
MAE/MSE 502 Spring 2020 HW1 Solution

Prob 1(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\, dx$, and $a_n = 2\int_0^1 P(x) \cos(n\pi x) dx$ for $n > 0$,

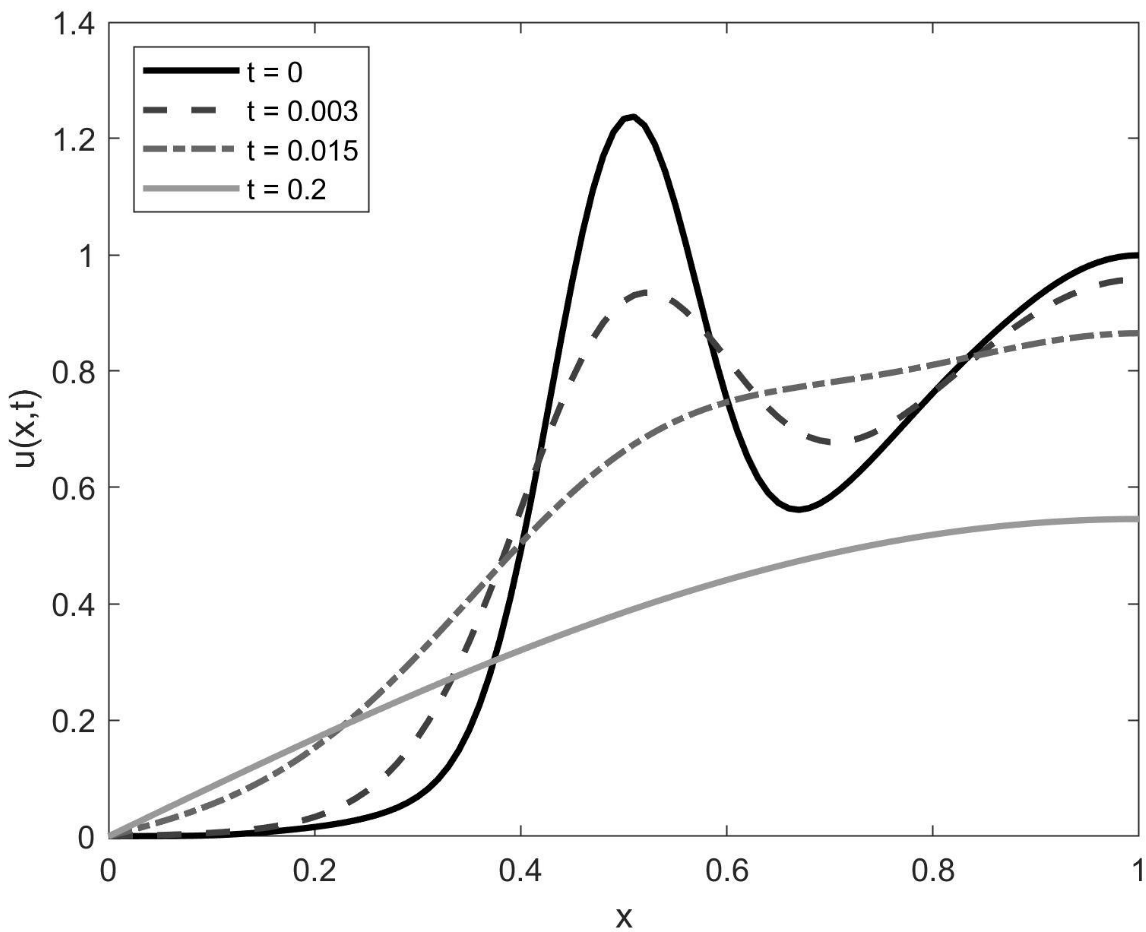


Prob 1(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi x}{2}) \exp(-(\frac{n\pi}{2})^2 t)$$

where the summation is over <u>odd values of *n* only</u>, and

$a_n = 2\int_0^1 P(x)\sin(\frac{n\pi x}{2})dx$, for <u>odd values of *n*</u>.



An example of Matlab code for making the plot for Prob 1(b) is given in the next page. In the code, the function mod(*M*, *N*) is the remainder of *M* divided by *N*. For example, mod(12, 5) = 2 since the remainder of 12 divided by 5 is 2. With that, mod(*n*,2) = 0 if *n* is even, and mod(*n*,2) = 1 if *n* is odd. This is used to eliminate all a_n with even values of *n*.

The code also illustrates how to make a clear black-and-white plot. With multiple lines, they can be distinguished by the line pattern and gray scale.

```
clear
x = [0:0.002:1]; t = [0 0.003 0.015 0.2];
p = 2*x.^3-x.^6+((1-cos(2*pi*x)).^10)/1024;
N = 50;
for n = 1:N
    if (mod(n,2) == 1)
        a(n) = trapz(x,sin(n*pi*x/2).*p)/trapz(x,sin(n*pi*x/2).^2);
    else
        a(n) = 0;
    end
end
for m = 1:length(t)
    u(m,:) = 0*x;
    for n = 1:N
        u(m,:) = u(m,:)+a(n)*sin(n*pi*x/2)*exp(-(((n*pi)/2)^2)*t(m));
    end
end
%
hold on
plot(x,u(1,:),'-','Color',[0 0 0],'LineWidth',2)
plot(x,u(2,:),'--','Color',[0.25 0.25 0.25],'LineWidth',2)
plot(x,u(3,:),'-.','Color',[0.4 0.4 0.4],'LineWidth',2)
plot(x,u(4,:),'-','Color',[0.6 0.6 0.6],'LineWidth',2)
xlabel('x'); ylabel('u(x,t)');
legend('t = 0','t = 0.003','t = 0.015','t = 0.2',...
       'Location','NorthWest')
box on
hold off
```

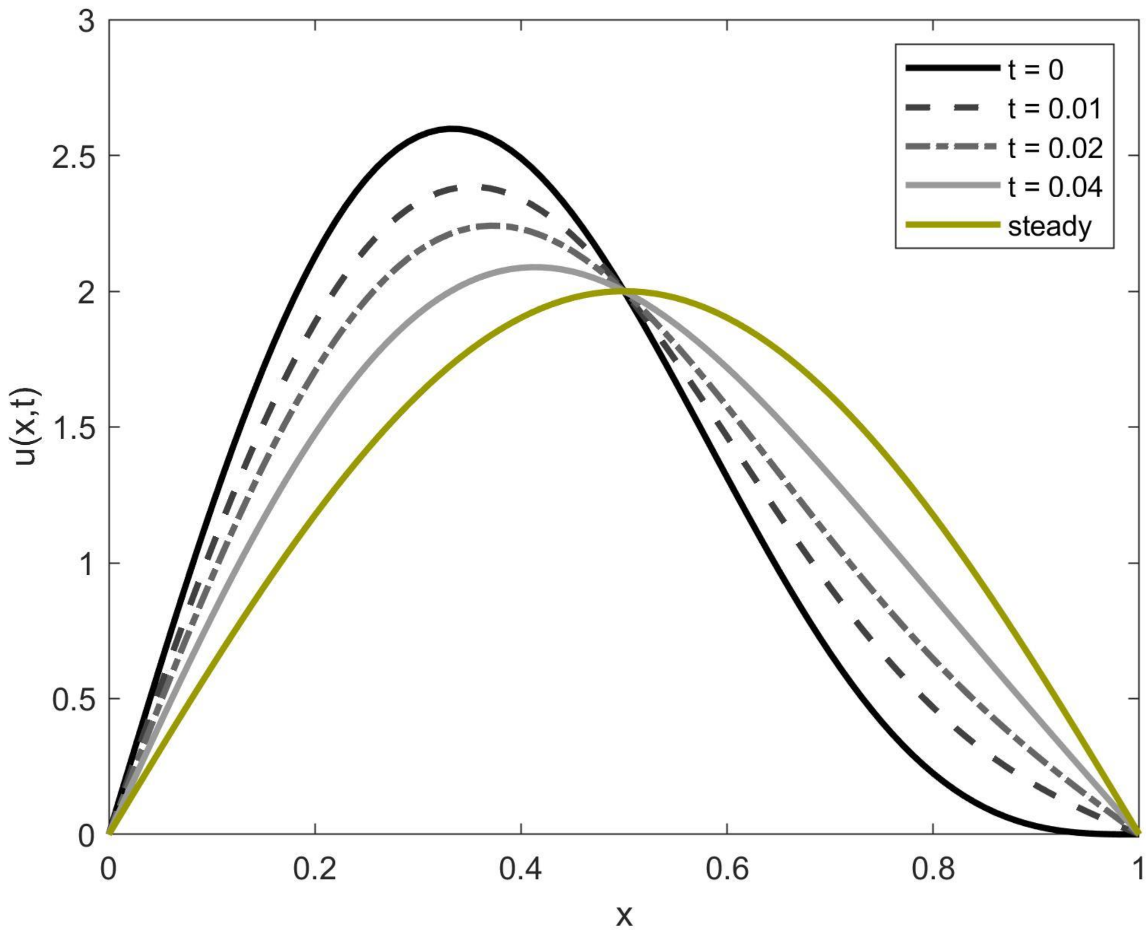
Prob 2

$$u(x,t) = 2\sin(\pi x) + \sin(2\pi x)\,(1+t)^{-3\pi^2}$$

Steady solution is

$$u_s(x) = 2\sin(\pi x)$$

Plot:



Prob 3

$$u(x,t) = e^{\sin(t)} + \cos(\pi x)\, e^{\sin(t) - \pi^2\left(1-e^{-t}\right)}$$

Prob 4

We will discuss the solution of this problem in class.