MAE/MSE 502, Spring 2020 HW2 Solution

Prob 1

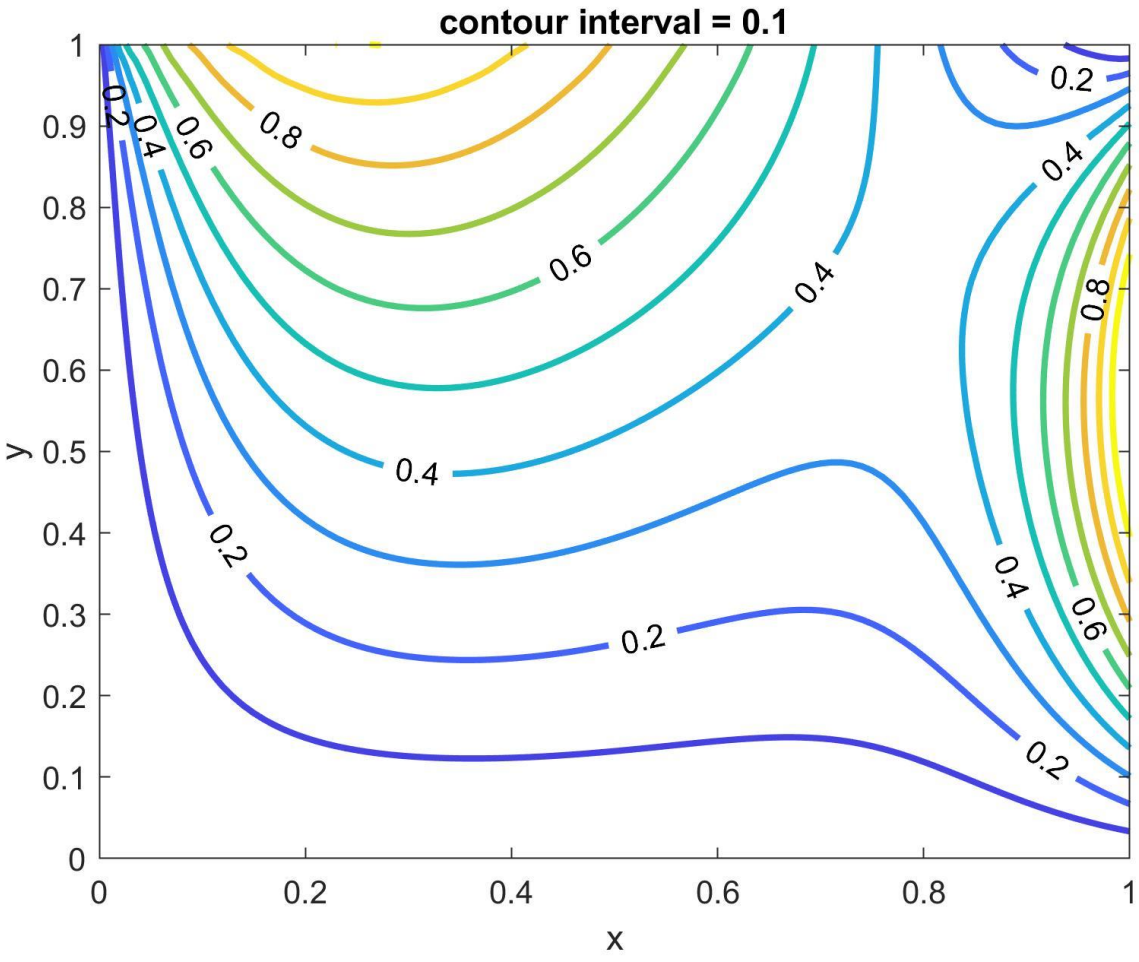
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y)\sinh(3n\pi x) + b_n \sin(n\pi x)\sinh\left(\frac{n\pi y}{3}\right)$$

where

$$a_n = \frac{2}{\sinh(3n\pi)} \int_0^1 3(y - y^3)\sin(n\pi y)\, dy$$

$$b_n = \frac{2}{\sinh(\frac{n\pi y}{3})} \int_0^1 \sin(\pi\sqrt{x})\sin(n\pi x) dx$$

Plot:



Prob 2

(a) Solvability condition is satisfied. There are infinitely many solutions

(b)

$$u(x,y) = C + 3y + \frac{\cosh(\pi y)\cos(\pi x)}{\pi \sinh(\pi)}$$

where *C* is an arbitrary constant (which corroborates the conclusion from Part (a)).

Prob 3

$$u(x,y) = \left(4 - \frac{4}{x^2}\right)\cos(y) + \left(\frac{64}{x^4} - x^2\right)\cos(3y)$$

Prob 4

$$u(x,y) = \cos(3\pi y) + y\cos(3\pi x) + [\cosh(4\pi y) - \tanh(4\pi)\sinh(4\pi y)]\cos(5\pi x)$$

The detail of the solution for Prob 2(b), 3, and 4 will be posted separately.