MAE/MSE 502, Fall 2020 HW3 Solution

Prob 1

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{8}\right)\cos\left(\frac{n\pi t}{8}\right)$$

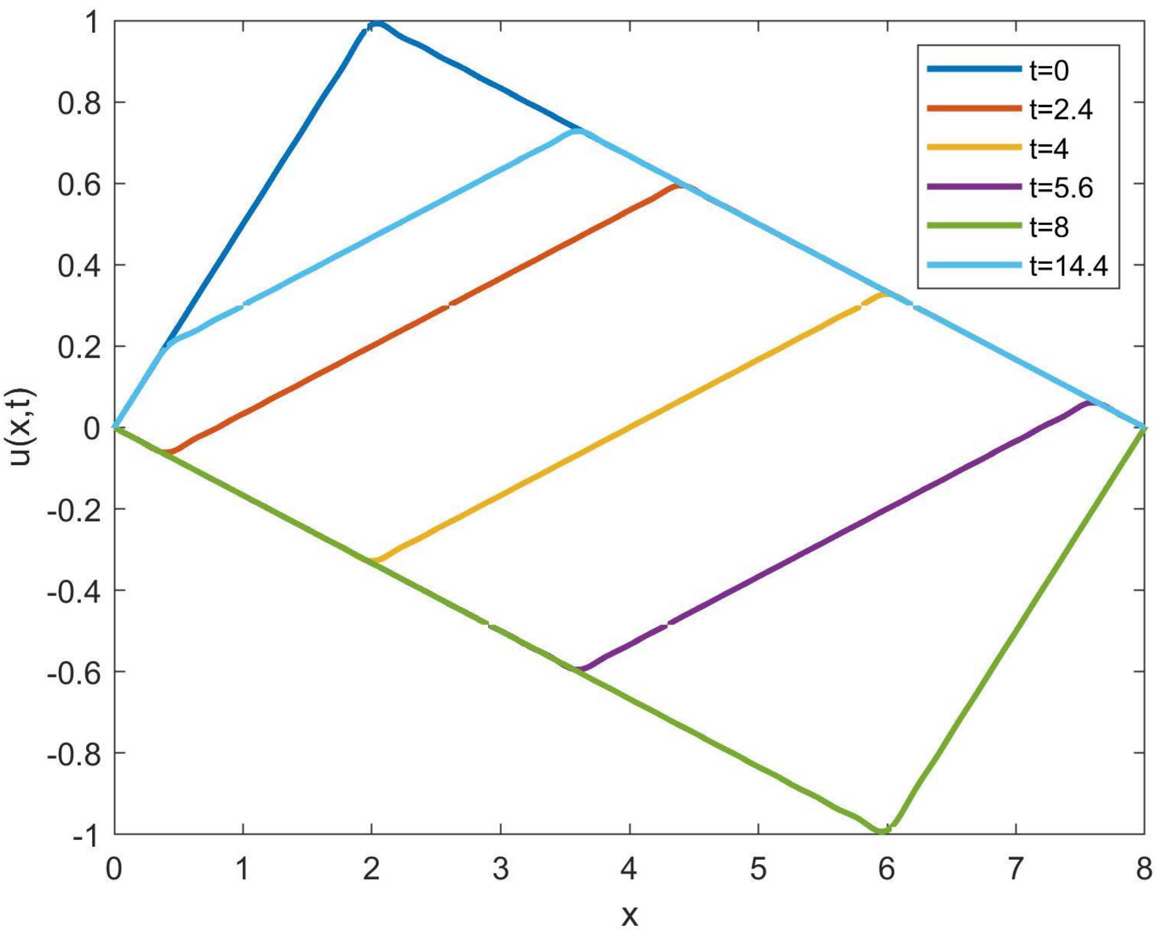
For Part (a):

$$a_n = \frac{1}{4}\left[\int_0^2 \frac{x}{2}\sin\left(\frac{n\pi x}{8}\right)dx + \int_2^8 \frac{(8-x)}{6}\sin\left(\frac{n\pi x}{8}\right)dx\right].$$

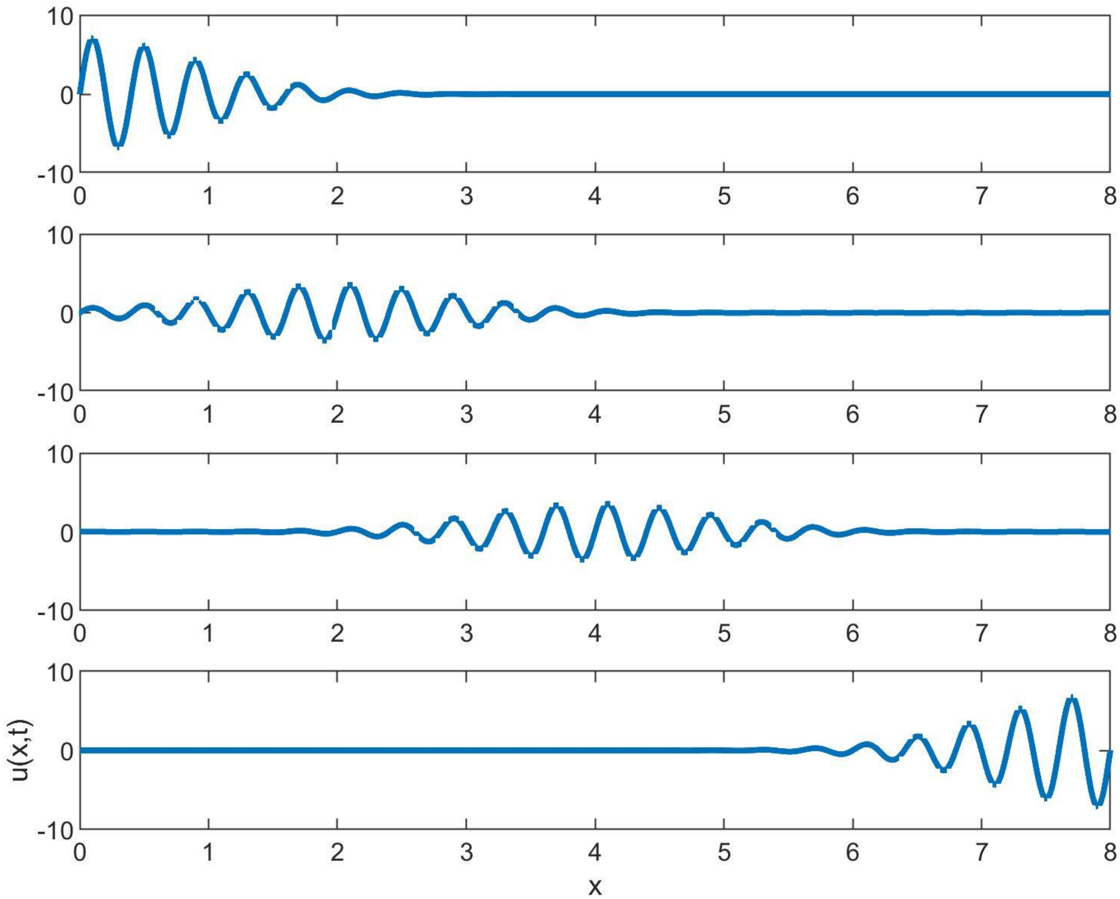
For Part (b):

$$a_n = \exp\left[-\left(\frac{n-40}{4}\right)^2\right] \text{, for } 30 \le n \le 50, \text{and } a_n = 0 \text{ otherwise}$$

Plot for (a)



Plot for (b): Top to bottom: $t = 0, 2, 4$, and 8



Prob 2

All $c > 0$ are eigenvalues. The corresponding eigenfunction is

$$G_c(x) = \frac{2 - 2\cosh(\sqrt{c})}{\sqrt{c}\,\sinh(\sqrt{c})}\cosh(\sqrt{c}\,x) + \frac{2}{\sqrt{c}}\sinh(\sqrt{c}\,x)$$

c = 0 is an eigenvalue. The corresponding eigenfunction is $G_0(x) = 2x + B$, where B is arbitrary.

All $c < 0$ but $c \neq -(n\pi)^2$, $n = 1, 2, 3, 4, \ldots$, are eigenvalues. The corresponding eigenfunction is

$$G_c(x) = -\frac{2 - 2\cos(\sqrt{-c})}{\sqrt{-c}\,\sin(\sqrt{-c})}\cos(\sqrt{-c}\,x) + \frac{2}{\sqrt{-c}}\sin(\sqrt{-c}\,x)$$

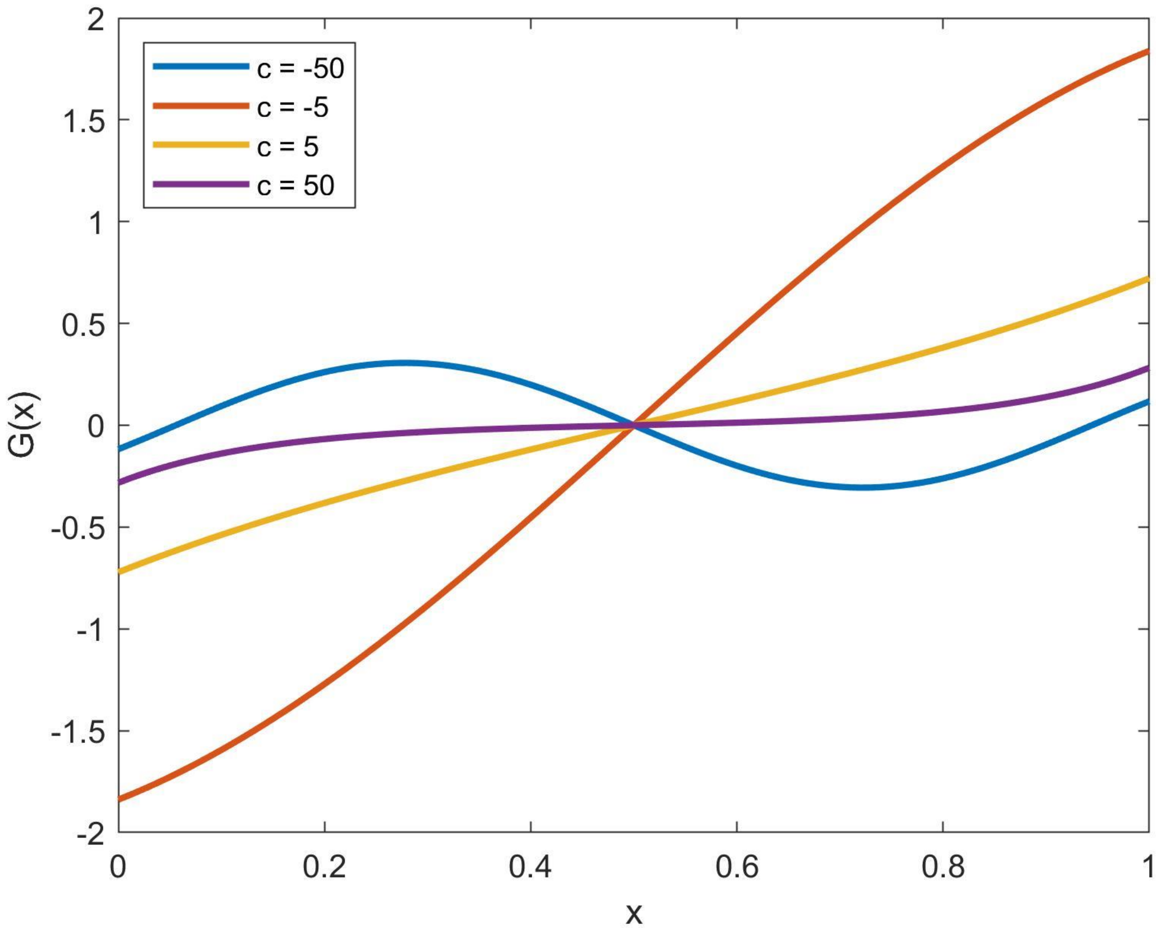
The special values of $c = -(n\pi)^2$, $n = 2, 4, 6, 8, \ldots$, are eigenvalues. The corresponding eigenfunction is

$$G_c(x) = A\cos(\sqrt{-c}\,x) + \frac{2}{\sqrt{-c}}\sin(\sqrt{-c}\,x), \qquad \text{where } A \text{ is arbitrary.}$$

The special values of $c = -(n\pi)^2$, $n = 1, 3, 5, 7, \ldots$, are NOT eigenvalues.

The orthogonality relation does not hold for the eigenfunctions.

Plot for Prob 2:



Prob 3

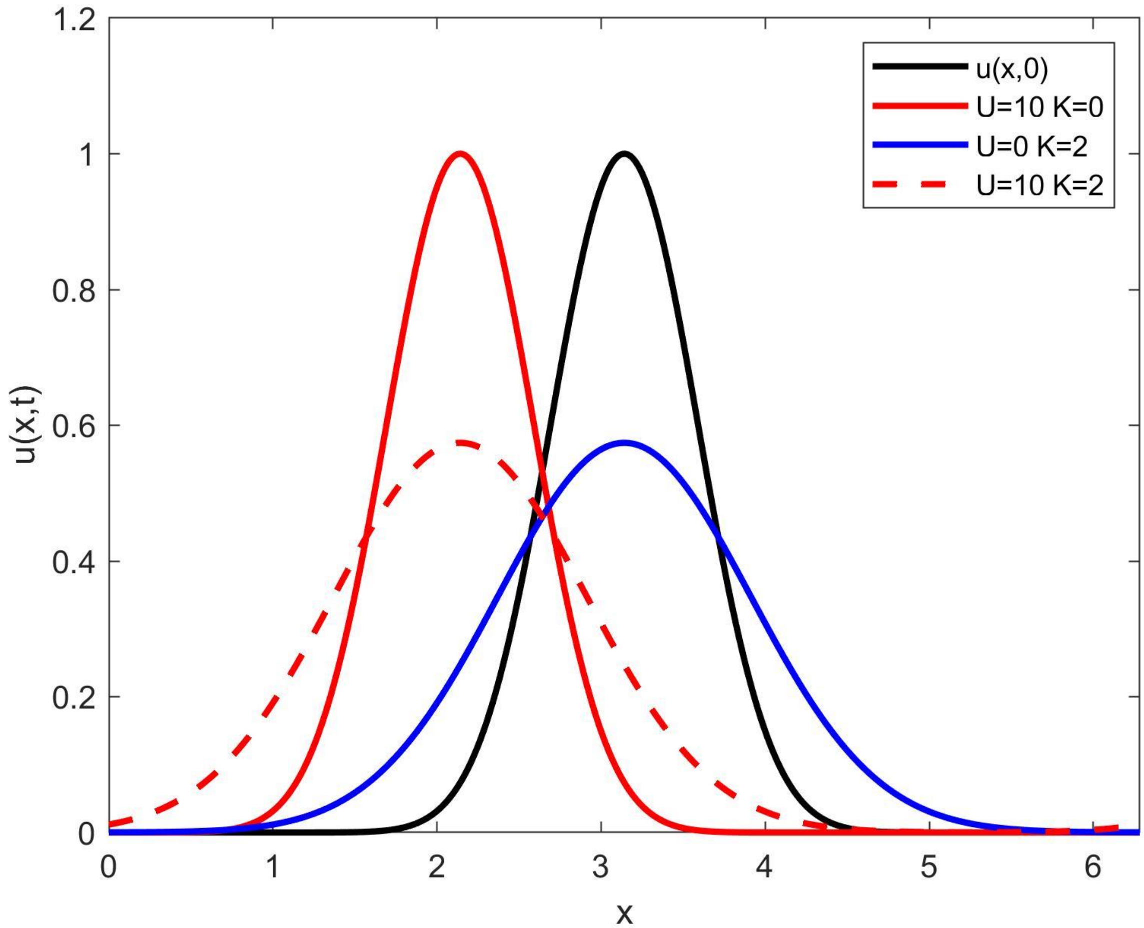
$$u(x,t) = \sum_{n=-\infty}^{\infty} C_n(0) e^{(inU - n^2 K)t + inx}$$

where

$$C_n(0) = \frac{1}{2\pi} \int_0^{2\pi} u(x,0) e^{-inx} dx$$

(Plot in next page)

Plot for Prob 3:



Prob 4

$$u(x,t) = (1+t)^4 + \sin(x)\,(1+t)^3 + \cos(2x + 24t)$$

Prob 5

$$u(x,t) = 2\,e^{4t} - 2\,e^{-t} + \sin(2x)$$