MAE/MSE 502, Spring 2020 Homework # 4

Please upload the report as a single pdf or doc/docx file to Canvas. The report should include a statement on collaboration. See the cover page of Homework #1 for the rules on collaboration.

For ALL problems in this homework, we expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be expressed in real numbers and functions.

Prob 1 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t\, e^{-\pi^2 t} \cos(\pi x) + 100\, e^{-t}$$

with the boundary conditions

(i) $u_x(0,t) = 0$ (ii) $u_x(1,t) = 0$ (iii) $u(x,0) = \cos(\pi x)$.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 4\, u$$

with the boundary conditions,

(i) $u_x(0,t) = 0$ (ii) $u(\pi,t) = 1$ (iii) $u(x,0) = \cos(2x) + \cos(0.5\, x)$

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 0.25\, u + 2020$$

with the boundary conditions,

(i) $u_x(0,t) = 0$ (ii) $u_x(\pi,t) = -1$ (iii) $u(x,0) = 2\cos(0.5x)$.

Note that the first two boundary conditions are imposed on the derivative of u.

Prob 4 (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 4u + 8 + t\sin(2x)$$

with periodic boundary conditions in the x-direction, and the boundary conditions in the t-direction given as

(i) $u(x,0) = 1$ (ii) $u_t(x,0) = \cos(x)$